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MONTEREY, CALIFORNIA

THESIS

SIGNAL SPACE MODELING AND ANALYSIS OF COMPLEX SPREADING MIMO SIGNAL WITH ORTHOGONAL COVERING IN INTERFERENCE AND FADING

by

Yit Peng Tan

September 2012

Thesis Advisor:	Ric Romero
Thesis Co-Advisor:	Tri Ha

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The modeling of a digital communication system using complex spreading multiple-input-multiple-output (MIMO) signal with orthogonal covering is discussed in this thesis. Specifically, we examine the MIMO signal that employs fixed and variable length orthogonal covering for both multiuser and multi-rate channels. The approach uses a discrete time formulation to model the signal for Rayleigh fading and various types of jamming interference. The use of variable length Walsh sequences, including concatenation of Walsh-data vectors for simultaneous multi-rate transmissions are explained. The derivations of the noise and interference covariances results are also discussed. In addition, the bit error performances for various jamming scenarios, including barrage, pulsed-noise and tone jamming, are analyzed.

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SIGNAL SPACE MODELING AND ANALYSIS OF COMPLEX SPREADING MIMO SIGNAL WITH ORTHOGONAL COVERING IN INTERFERENCE AND FADING

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Submitted in partial fulfillment of the
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ABSTRACT

The modeling of a digital communication system using complex spreading multiple-input-multiple-output (MIMO) signal with orthogonal covering is discussed in this thesis. Specifically, we examine the MIMO signal that employs fixed and variable length orthogonal covering for both multiuser and multi-rate channels. The approach uses a discrete time formulation to model the signal for Rayleigh fading and various types of jamming interference. The use of variable length Walsh sequences, including concatenation of Walsh-data vectors for simultaneous multi-rate transmissions are explained. The derivations of the noise and interference covariances results are also discussed. In addition, the bit error performances for various jamming scenarios, including barrage, pulsed-noise and tone jamming, are analyzed.

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LIST OF ACRONYMS AND ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BPSK	Binary Phase Shift Key
CDM	Code Division Multiplexing
CDMA	Code Division Multiple Access
DSSS	Direct Sequence Spread Spectrum
FH	Frequency Hopping
ISM	Industrial, Scientific and Medical
MAI	Multiple Access Interference
MIMO	Multiple-Input-Multiple-Output
MRC	Maximal Ratio Combining
OSTBC	Orthogonal Space-Time Block Code
OVSF	Orthogonal Variable Spreading Factor
PN	Pseudo-noise
PSD	Power Spectral Density
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Key
RF	Radio Frequency
SISO	Single-Input-Single-Output
SJR	Signal-to-jamming Ratio
SJNR	Signal-to-jamming-noise Ratio
SNR	Signal-to-noise Ratio
SS	Spread Spectrum

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EXECUTIVE SUMMARY

Traditional communications systems in a fading environment have been a research area for many years. Techniques such as spatial diversity in the receiver are commonly used in modern communications. Recently, the use of multiple antennas at both the transmitter and receiver has shown performance improvement over a fading channel. Interference such as jamming is always a practical problem, especially, in military communication systems. Jamming is often mitigated by the use of spread spectrum (SS) techniques. Such techniques are also employed in civilian systems for multiuser access such as code-division multiple access (CDMA) and the second-generation (2G) cellular standard IS-95. Orthogonal covering, which uses fixed and variable length Walsh sequences, has been known to provide orthogonality and enables channeling with different data rates. A system employing Multiple-Input-Multiple-Output (MIMO), complex spreading and orthogonal covering may improve the system performance; hence, there is a need to investigate how such a system performs in a fading channel with interference.

The objective of this thesis is to discuss and analyze the modeling of a complex spreading MIMO system with orthogonal covering in Rayleigh fading and interference. The concept of the system is illustrated in Figure 1.

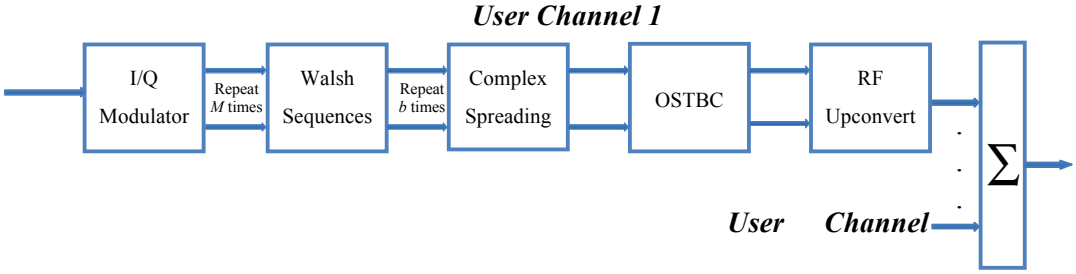

Figure 1. Integrated spreading MIMO system with orthogonal covering procedure and orthogonal space-time block codes (OSTBC).

First, the complex symbol from each channel is repeated M times and coded with the Walsh sequence of spreading factor M. After that, the Walsh vector is repeated b times to meet the common chip rate of the PN code with spreading factor N. The PN code is chosen such that $N = bM$. The integrated spread signal is then coded with the OSTBC

before transmission. Different channels are assigned unique Walsh codes. For multi-rate channels, the channel with the higher data rate is assigned with a shorter code, and a lower data rate channel is assigned with a longer code. Hence, in order to meet the common chip rate for an orthogonal covering process, the vectors from the higher data rate channel must be concatenated. Orthogonal covering is achieved by linearly combining the integrated spread signals from each channel.

When the signal goes through an AWGN channel with interference, the received signal is corrupted with noise and jamming. The covariances of the noise and jamming are calculated using a discrete time formulation. The results are used to derive the bit error rate (BER) performance for a 32-quadrature amplitude modulation (QAM) system for each of the three jamming scenarios: barrage, pulsed-noise and tone jamming.

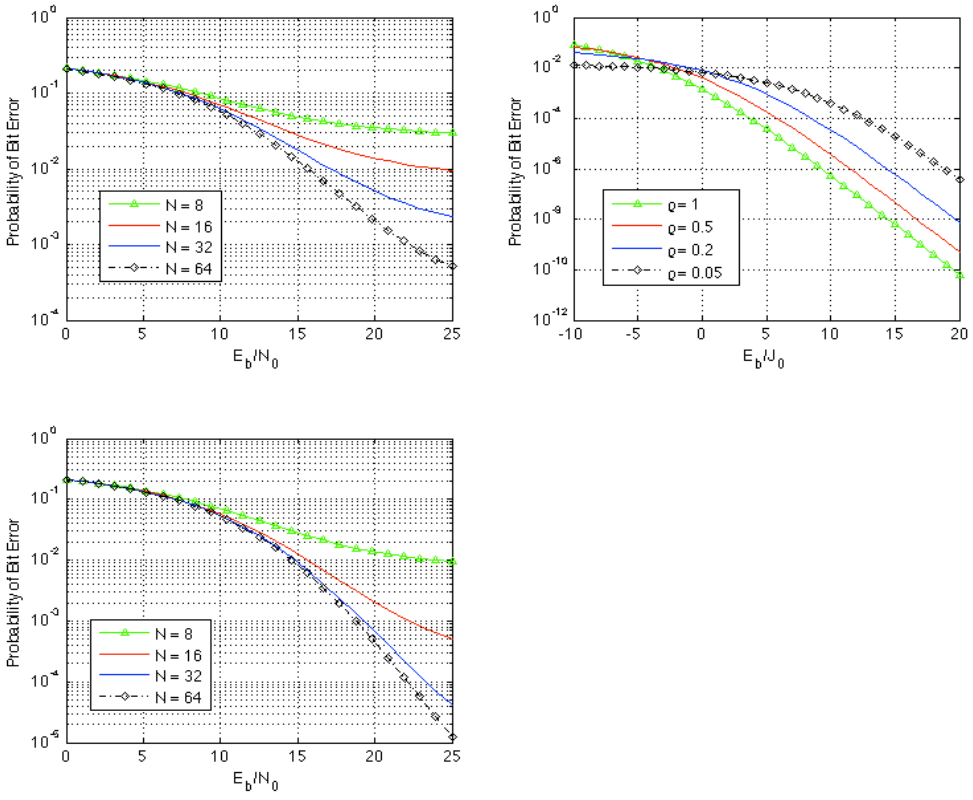

Figure 2. BER curves are shown for the following: a) barrage Jamming (MIMO (2×2) with $E_b/J_0 = 0$ dB and spreading ($N = 8, 16, 32, 64$)), b) pulsed-noise jamming (MIMO (2×2) with $N = 64$ and increasing ρ), and c) tone jamming (MIMO (2×2) with $A_j/A = 0$ dB and spreading ($N = 8, 16, 32, 64$)).

The BER curves for each jamming scenario for fixed transmission rate channel are shown in Figure 2. The BER improves when the PN spreading factor N increases. This confirms the advantage of spreading against jamming, where the power of the jamming is decreased in proportion to the spreading gain. In Figure 2, E_b/N_0 is energy per bit to noise power spectral density ratio, E_b/J_0 is energy per bit to jammer power spectral density ratio and A_j/A is the jammer amplitude to carrier amplitude.

For a multi-rate channel, the spreading factor for each symbol is reduced due to the concatenation of the vectors. It is clear that the BER is degraded due to the lower effective spreading gain. The results are illustrated in the BER curves in Figure 3, where it is seen that the BER performance degrades for all three jamming scenarios as the transmission rate R_s increases.

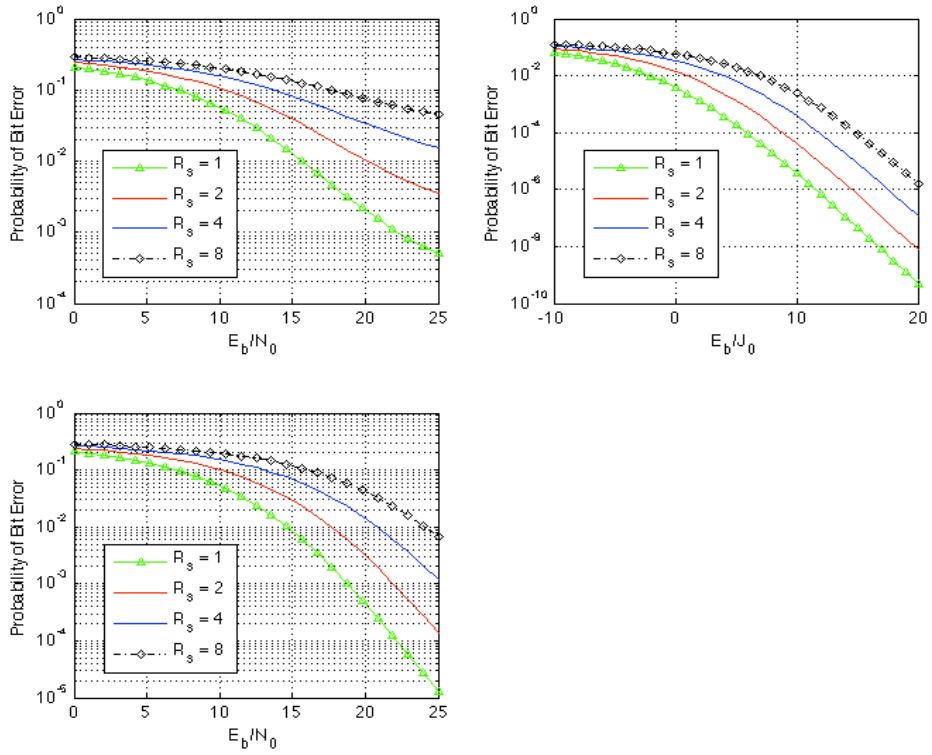

Figure 3. BER curves are shown for: a) barrage Jamming (MIMO (2×2) with $E_b/J_0 = 0$ dB, $N = 64$ and increasing R_s), b) pulsed-noise jamming (MIMO (2×2) with $N = 64$, $\rho = 0.5$ and increasing R_s), and c) tone jamming (MIMO (2x2) with $A_j/A = 0$ dB, $N = 64$ and increasing R_s).

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I. INTRODUCTION

A. BACKGROUND

In a traditional communication system, signals received may be degraded or lost due to fading or/and interference. Techniques such as spatial diversity are often used in a fading environment to improve the performance of a wireless system. Multiple-input-multiple-output (MIMO) techniques provide great improvement to the performance over fading channels. Spread spectrum techniques are well known for mitigating interference in a non-fading environment. In military communications, interference such as jamming is a practical problem. The interference may be caused by deliberate jamming by an adversary. On other hand, commercial systems such as IS-95 and CDMA2000 employ orthogonal covering techniques for channel separation between users with different data rates to mitigate interference such as multiple access interference (MAI) [1]. Thus, a conventional system experiencing both interference and fading may suffer from severely degraded bit error rate (BER) performance. For these channel problems, it is clear that MIMO, spread spectrum, and orthogonal covering techniques can be used together to achieve better performance and provide orthogonality for multi-rate user transmission. Hence, there is a need to find out how a complex spreading MIMO system with orthogonal covering performs in a fading channel with interference.

B. OBJECTIVE

The objective of this thesis is to derive and analyze the performance of a system employing spatial diversity, complex spread spectrum, and orthogonal covering in Rayleigh fading and jamming interference.

C. RELATED WORK

The MIMO signaling with spread spectrum techniques have commonly been used over the years to combat fading and other varied interference. The studies in [2] utilize the discrete-time approach to model the complex spreading MIMO systems in Rayleigh fading and interference. The BER performance evaluation results are covered in both [2] and [3]. A theoretical explanation of complex spreading with Walsh functions is covered

in [1]. In this thesis, orthogonal covering techniques are applied to the previous work done in [2]. In this work, we present the signal modeling, analyses, and performance results.

D. SCOPE AND ORGANIZATION

In Chapter II, the signal model background is discussed. The techniques in modeling the complex spreading MIMO signal with orthogonal covering are explained. These include spread spectrum (SS), orthogonal covering, orthogonal space-time block codes (OSTBC), fading channel, and maximal ratio combining (MRC).

In Chapter III, the signal model for the complex spread spectrum signaling with orthogonal covering is developed. A detailed explanation of the procedures for complex spreading and de-spreading with orthogonal covering is given.

In Chapter IV, the procedures for multi-rate transmission channels, including the concatenation of Walsh codes are covered.

In Chapter V, MIMO and MRC are discussed, and the noise and interference covariances of the signals are derived.

In Chapter VI, BER results are presented.

In Chapter VII, conclusions are summarized.

II. CONCEPT BACKGROUND

In this chapter, the techniques used in the signal space modeling of the system are examined. The fundamentals of spread spectrum, orthogonal covering, space-time block codes and maximal ratio combining are covered.

A. INTRODUCTION

Over the years, many techniques have evolved to improve the overall performance of a digital communications system. For example, spread spectrum techniques are widely used in military wireless applications due to their effectiveness against interference. Alamouti [4] has invented a simple transmit diversity technique which significantly improves the bit error rate performance without additional bandwidth expense. Spatial diversity techniques such as MIMO schemes have the ability to overcome multipath propagation, which is traditionally a pitfall of wireless transmission. Orthogonal codes such as Walsh–Hadamard code [5] are used in CDMA cellular communications to separate different users' channels. Orthogonal covering techniques make use of variable Walsh functions to support simultaneous multi-rate transmission over a single carrier without incurring interference between users. A typical communication system with spreading block diagram is shown in Figure 1. In this chapter, techniques that are used to model the complex spreading and de-spreading of MIMO signal with orthogonal covering are studied.

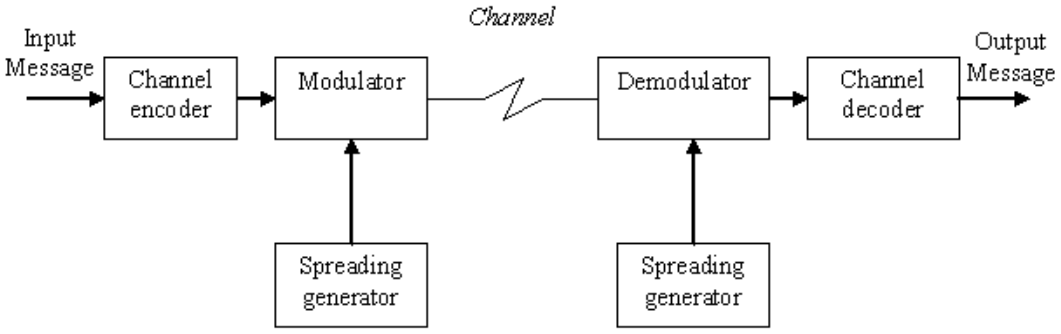

Figure 1. Block diagram of the spread spectrum communication system is shown (From [6]).

B. SIGNAL SPACE MODELING

1. Spread Spectrum Signaling

Spread spectrum signaling is a technique that spectrally spreads the signal of the transmitted message. The SS modulated signal has a bandwidth much larger than the bandwidth of the narrowband information signal [1]. The main purpose of SS is to protect the signal's integrity by reducing the effectiveness of a jammer. In order to jam a SS signal, the jamming signal power has to be distributed over a larger spread of bandwidth. When a jammer in the channel is added to the SS received signal, at the receiver the interference jammer energy is spread out during de-spreading. This results in an effective reduction of the magnitude of the power spectral density (PSD) of the jamming signal. The use of spread spectrum techniques has a history dating back to World War II during developments of radar and ranging techniques [7]. The characteristics such as low PSD, high immunity to jamming and interference, high resolution ranging and the possibility of code-division multiple access have made SS pervasive in military applications. In 1985, the Federal Communications Commission made a decision to allow the use of SS signals in industrial, scientific and medical (ISM) bands (902–928 MHz, 2400–2483.5 MHz, and 5725–5850 MHz) at power levels set at 1W [8]. There are two types of spread spectrum modulated signals - direct sequence (DS) and frequency hopping (FH). In this thesis, DS signaling is used.

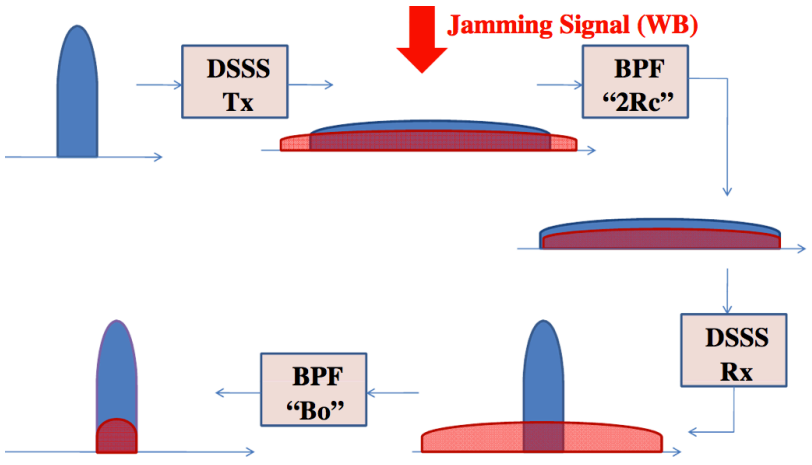

Figure 2. Effect of direct sequence spread spectrum in jamming is shown (From [9]).

a. Direct Sequence (DS) Modulation

Direct sequence spread spectrum (DSSS) is a SS technique that spreads a data signal by combining with a higher data-rate bit sequence or chipping code. The chipping code increases the signal's resistance to jamming interference. The effect of jamming in a direct sequence spread spectrum is illustrated in Figure 2.

In DSSS, the DS signal is generated by encoding the binary data 0s and 1s of bit rate $R_b = 1/T_b$ with a periodic pseudo-noise (PN) binary-valued sequence of N chips. The pseudo-random code with chip rate R_c is locally generated, where $R_c = 1/T_c = NR_b$, and the sequence period is the same as the bit time. To transmit a 0 bit, the transmitter sends the PN sequence; to transmit a 1 bit, the transmitter sends the complement PN sequence. This operation can be implemented by repeating a data bit N times and modulo-2 adding them to the PN sequence. Since the chip rate is N times the bit rate, the spread bandwidth is N times larger than the corresponding narrowband signal. The spreading factor N is often referred to as the processing gain of the spread spectrum signal. It may be dependent on the modulation and the demodulation, as well as the type of jamming signal [1]. The longer PN codes (the larger N, the better it is against jamming) come with a trade-off between bandwidth and capability.

An illustration of spread spectrum signaling is shown in Figure 3. It can be seen from the top diagram (shown in the frequency domain) that the spread bandwidth B_c is much larger than the original signal bandwidth B_s. This corresponds to the much shorter time period T_c of the spreading code as compared to the original symbol period T_s. The spread signals may seem to appear as noise in the frequency domain. The wide bandwidth provided by the PN sequence allows the modulated signal power to drop below the noise threshold without loss of information.

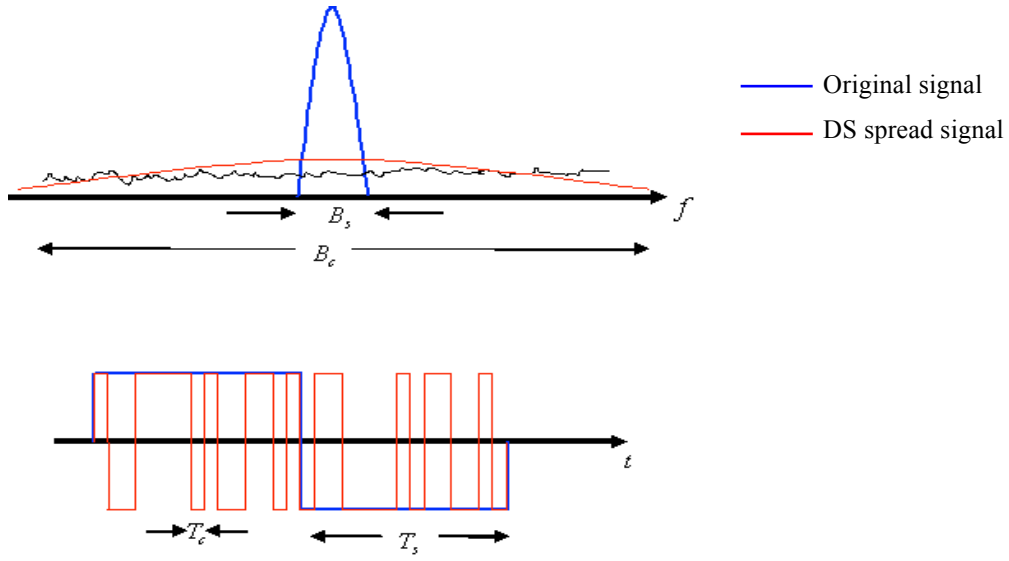

Figure 3. Direct sequence spread spectrum signaling is shown (From [6]).

One example of a DS modulator is shown in Figure 4. The DS-PSK signal is generated by modulating the PN code onto the information signal using phase-shift keying i.e., binary-phase shift key (BPSK), quadrature phase shift key (QPSK), and so on. Then the signal is radio frequency (RF) upconverted with a RF carrier.

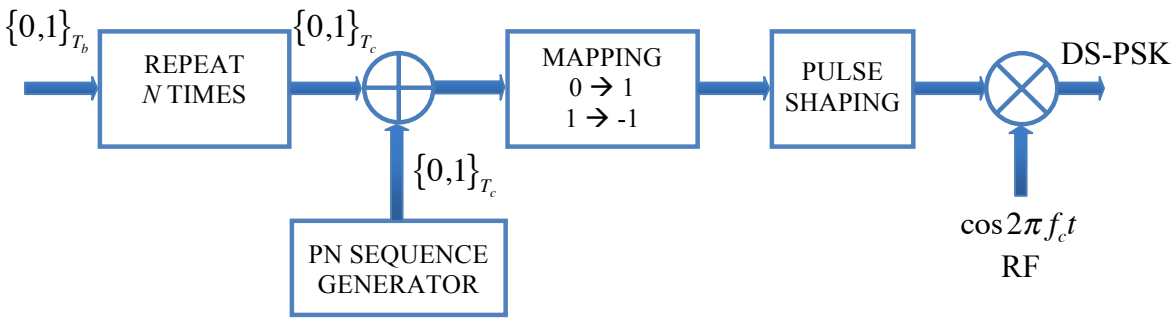

Figure 4. Direct sequence-phase shift keying transmitter (From [1]).

To show an example, consider a PN sequence code $\mathbf{c} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$ (in vector format) which has a spreading factor $N = 4$. The data bit '0' is to be encoded. The data bit is repeated N times to provide the data sequence $\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$. To spread the signal, the two sequences are modulo-2 added, and thus, the spread signal is

given by $\mathbf{x} = \mathbf{c} \oplus \mathbf{d} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$. Applying the mapping $0 \to 1$ and $1 \to -1$, we get the sequence $\mathbf{x} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$. This is the sequence that is sent when the data bit is 0. When the data bit is 1, the spread sequence is $\mathbf{x} = \begin{bmatrix} -1 & 1 & -1 & 1 \end{bmatrix}$, which is the complement of the previous sequence. Alternatively, the spread sequence **x** can also be generated via multiplication if we apply the mapping on the two sequences ***c*** and ***d***, and then multiply the corresponding elements in ***c*** and ***d*** to get the spread signal **x**.

b. Complex Spreading Signaling

In complex spreading [1], the signal is spread via the I and Q (in-phase and quadrature, respectively) channels. The data symbols in I and Q channels are spread with a complex spreading function. Given an arbitrary complex symbol $\boldsymbol{d} = d + j\hat{d}$, d is the input-I bit and $\hat{d}$ is the input-Q bit. The complex symbol is spread with a PN code, $\boldsymbol{c} = \boldsymbol{c}_I + j\boldsymbol{c}_Q$, and the complex spread symbol $\boldsymbol{s}$ is given by $\boldsymbol{s} = \boldsymbol{dc}$, where it takes the form $\boldsymbol{s} = \boldsymbol{s}_I + j\boldsymbol{s}_Q$. Thus, the complex spreading I and Q symbols can be expressed as

$$\begin{aligned} \boldsymbol{s} = \boldsymbol{s}_I + j\boldsymbol{s}_Q &= \left(d + j\hat{d}\right)\left(\boldsymbol{c}_I + j\boldsymbol{c}_Q\right) = \left(d\boldsymbol{c}_I - \hat{d}\boldsymbol{c}_Q\right) + j\left(\hat{d}\boldsymbol{c}_I + d\boldsymbol{c}_Q\right) \\ \therefore \boldsymbol{s}_I &= d\boldsymbol{c}_I - \hat{d}\boldsymbol{c}_Q, \quad \boldsymbol{s}_Q = \hat{d}\boldsymbol{c}_I + d\boldsymbol{c}_Q. \end{aligned} \tag{2.1}$$

Hence, the name complex spreading is given. A diagram of a complex spreading procedure is shown in Figure 5.

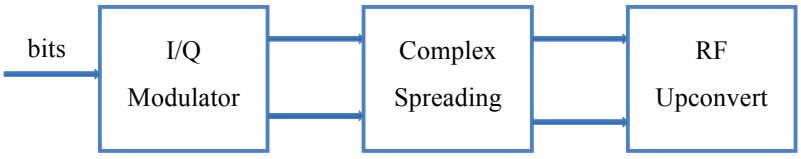

Figure 5. Complex spreading procedure is illustrated (After [2]).

2. Orthogonal Covering

Orthogonal covering or code-division multiplexing (CDM) is a technique that employs orthogonal functions, such as Walsh functions, to multiplex different user bit streams onto a single carrier without interference between user data [1]. It also employs variable length orthogonal codes to support simultaneous multi-rate transmission.

a. Walsh-Hadamard codes

Walsh-Hadamard orthogonal codes have been used in numerous communication systems during the last decade [5]. They are used in DS communication systems such as IS-95 and CDMA2000 [1], [5]. They have been known since 1923 and are advantageous because they assume only values of ±1 and are easily generated by digital circuits. A Walsh code is simply a row or column taken from a Hadamard matrix. A Hadamard matrix is a symmetric square matrix composed of 1s and 0s with a dimension that is a power of two and is given by (after mapping $0 \to 1$, $1 \to -1$):

$$\mathbf{H}_1 = \begin{bmatrix} 1 \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \cdots, \mathbf{H}_m = \begin{bmatrix} \mathbf{H}_{m/2} & \mathbf{H}_{m/2} \\ \mathbf{H}_{m/2} & -\mathbf{H}_{m/2} \end{bmatrix}, \tag{2.2}$$

where an 8×8 Hadamard matrix is

$$\mathbf{H}_8 = \begin{bmatrix} \mathbf{H}_4 & \mathbf{H}_4 \\ \mathbf{H}_4 & -\mathbf{H}_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}. \tag{2.3}$$

In an 8×8 Hadamard matrix, any two rows or columns are mutually orthogonal. Thus, Walsh codes, which are given by the rows of the Hadamard matrix, are simply orthogonal codes. This also means that the cross-correlation between two different Walsh codes is zero when aligned. A Walsh coded signal appears as random noise unless the receiver

uses the same code as the one used at the transmitter. Orthogonal Walsh functions are defined in order of N as $W_N = \{w_j(t); t \in (0,T),\ j = 0,1,\text{K}\ ,N-1,$ consisting of $N = 2^k$ elements that are functions of time with following properties [10]:

- $w_j(t)$ takes on the values $\{+1,-1\}$ except at a finite number of points of discontinuity, where it is defined as zero.
- $w_j(t) = 1$ for $j = 0$.
- $w_j(t)$ has precisely j sign changes in the interval $(0,T)$.
- $\int_0^T w_j(t) w_k(t)\, dt = \begin{cases} 0, & if\ j \neq k \\ T, & if\ j = k \end{cases}$ (Orthogonality).

To explain the fourth property (orthogonality), we take two Walsh codes from the row of the 8×8 Hadamard matrix given by $\boldsymbol{w}_2 = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$ and $\boldsymbol{w}_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix}$. The inner product of the two functions yields $\langle \boldsymbol{w}_2, \boldsymbol{w}_5 \rangle = 0$. A set of Walsh function of order eight is illustrated in Figure 6.

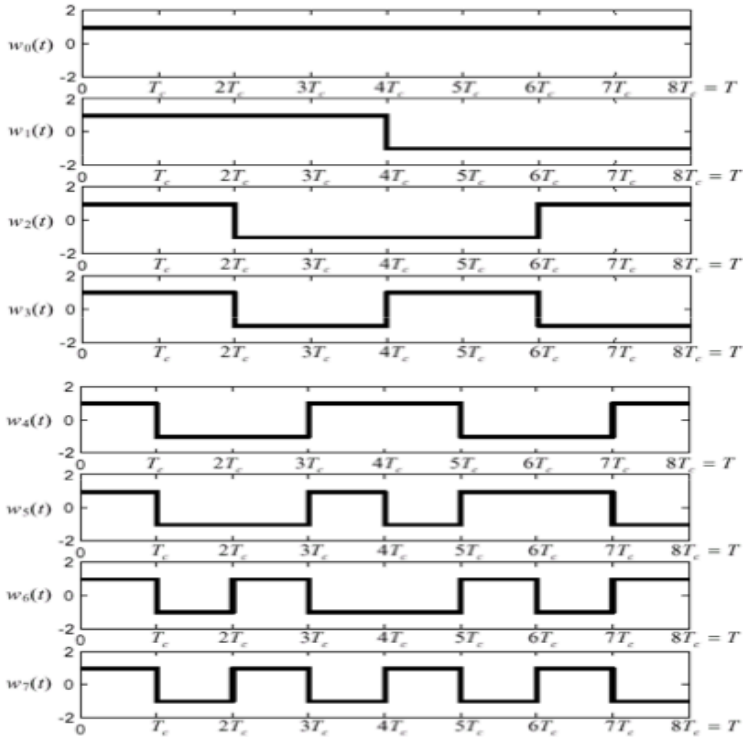

Figure 6. A set of Walsh function of order eight is shown (From [10]).

b. Orthogonal Covering using Walsh Functions

Orthogonal Walsh functions are used in practice when employing orthogonal covering techniques. The procedure is similar to the PN spreading sequence. Each user channel is assigned a Walsh sequence of length M with chip rate $R_w = MR_b$, where R_b is the bit rate. The user bit is repeated M times and then modulo-2 added to the Walsh chips. Then mapping is applied to the resulting Walsh symbol. A set of M Walsh functions can accommodate up to M user channels. The orthogonal covering is achieved by summing the symbols of M users together. A concept of orthogonal covering is illustrated in Figure 7.

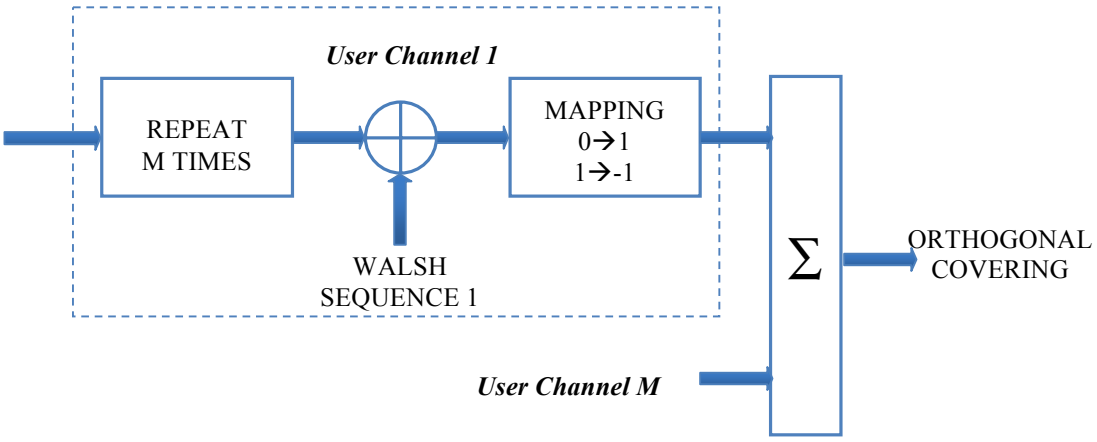

Figure 7. Orthogonal covering procedure is illustrated (After [1]).

c. Orthogonal Covering with Variable Length Sequences

Variable length orthogonal sequences, also known as orthogonal variable spreading factors (OSVF), are designed to improve the capability of a system accommodating different transmission bit rates. The OSVF code is generated by using a Walsh-Hadamard matrix. Each OVSF code is represented by $\boldsymbol{w}_M^{(i)}$, where M denotes the spreading factor (or "chips per bit") and i denotes the branch number. In practice, it is common for the spread bandwidth to be the same for all users in the system. Therefore, for multi-rate transmission, variable spreading factors codes are required. Higher data rates are assigned a shorter length code and lower data rates are assigned a longer length code so that they meet the same chip rate and spread bandwidth. The variable length orthogonal codes are described by using the tree structure as shown in Figure 8.

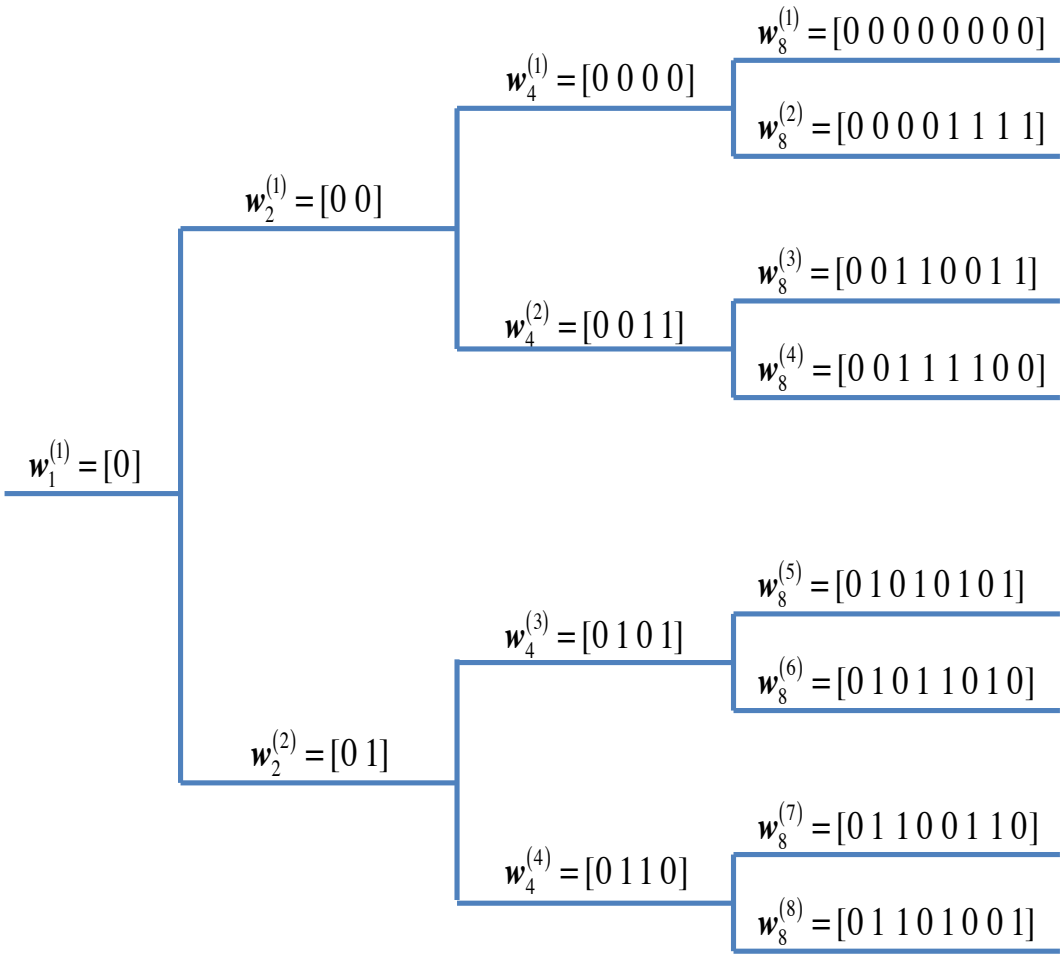

Figure 8. Code tree for variable length orthogonal sequences is generated (From [11]).

Now, we describe the code tree procedure. Start at the root of the tree with Walsh code $w_1^{(1)} = 0$. Move to the subsequent branches to create sequences with twice the spreading factor. Every move to the next branch to the right doubles the data rate. The upper branch code contains the cascade of two parent code sequences. The lower branch code contains the cascade of the parent code sequences and its complement. Variable length orthogonal sequences cannot come from the same branch. The generated codes of the same branch constitute a set of orthogonal Walsh functions. Any two codes of different branches are also orthogonal except for the case when one of the two codes is a

mother code of the other. Furthermore, if a code of any branch is assigned to a user, any codes generated from that code cannot be assigned to other users of the same bandwidth requesting lower rates.

An example to illustrate orthogonal covering using two channels [1], one with bit rate R_b (channel one) and the other with bit rate $2R_b$ (channel two) follows. Channel one employs a Walsh sequence $\boldsymbol{w}_8^{(3)}$ of eight chips per bit; therefore, the chip rate is $8R_b$. To meet the requirement of common chip rate, channel two has to employ a Walsh sequence $\boldsymbol{w}_4^{(3)}$, which has four chips per bit and is orthogonal to $\boldsymbol{w}_8^{(3)}$. Hence, the chip rate of channel two is $2R_b \times 4 = 8R_b$. The data bit for channel one is '1' and channel two is '0,1.' The data bit is repeated eight times for channel one and four times for channel two. The sequences are given by $\boldsymbol{d}$:

$$\boldsymbol{d}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix};\ \boldsymbol{d}_{2,0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{d}_{2,1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \tag{2.4}$$

and the Walsh sequences are given by

$$\boldsymbol{w}_8^{(3)} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix},\ \boldsymbol{w}_4^{(3)} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}. \tag{2.5}$$

The resulting Walsh symbols in each channel are derived via modulo-2 addition given by

$$\begin{aligned} \boldsymbol{s}_1 &= \boldsymbol{d}_1 \oplus \boldsymbol{w}_8^{(3)} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \\ \boldsymbol{s}_{2,0} &= \boldsymbol{d}_{2,0} \oplus \boldsymbol{w}_4^{(3)} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix},\ \boldsymbol{s}_{2,1} = \boldsymbol{d}_{2,1} \oplus \boldsymbol{w}_4^{(3)} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}. \end{aligned} \tag{2.6}$$

Applying the mapping $0 \to 1$ and $1 \to -1$ to the Walsh symbols, we get

$$\boldsymbol{s}_1 = \begin{bmatrix} -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \end{bmatrix},\ \boldsymbol{s}_{2,0} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix},\ \boldsymbol{s}_{2,1} = \begin{bmatrix} -1 & 1 & -1 & 1 \end{bmatrix}. \tag{2.7}$$

Walsh symbols of channel two (to achieve the chip rate of channel one) are concatenated yielding

$$\boldsymbol{s}_2 = \left(\boldsymbol{s}_{2,0}, \boldsymbol{s}_{2,1}\right) = \begin{bmatrix} 1 & -1 & 1 & -1, & -1 & 1 & -1 & 1 \end{bmatrix}. \tag{2.8}$$

The orthogonal covering is achieved by linearly combining the Walsh symbols $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ as given by

$$\boldsymbol{s} = \boldsymbol{s}_1 + \boldsymbol{s}_2 = \left[\boldsymbol{s}_{11}, \boldsymbol{s}_{12}\right] = \left[\begin{array}{cccc} 0 & -2 & 2 & 0, \end{array}\right. \left.\begin{array}{cccc} -2 & 0 & 0 & 2 \end{array}\right]. \tag{2.9}$$

To recover the data, we calculate the normalized inner products using the Walsh sequences $\boldsymbol{w}_8^{(3)} = \left[\begin{array}{cccccccc} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{array}\right]$ and $\boldsymbol{w}_4^{(3)} = \left[\begin{array}{cccc} 1 & -1 & 1 & -1 \end{array}\right]$. The results are given by

$$\begin{aligned}
\left\langle \boldsymbol{s}, w_8^{(3)} \right\rangle \Big/ 8 &= \frac{1}{8}\left\{\begin{array}{l} (0\times 1)+(-2\times 1)+(2\times -1)+(0\times -1)+(-2\times 1)+ \\ (0\times 1)+(0\times -1)+(2\times -1) \end{array}\right\} \\
&= \frac{1}{8}(-8) = -1 \\
\left\langle \boldsymbol{s}_{11}, w_4^{(3)} \right\rangle \Big/ 4 &= \frac{1}{4}\left\{(0\times 1)+(-2\times -1)+(2\times 1)+(0\times -1)\right\} \\
&= \frac{1}{4}(4) = 1 \\
\left\langle \boldsymbol{s}_{12}, w_4^{(3)} \right\rangle \Big/ 4 &= \frac{1}{4}\left\{(-2\times 1)+(0\times -1)+(0\times 1)+(2\times -1)\right\} \\
&= \frac{1}{4}(-4) = -1.
\end{aligned} \tag{2.10}$$

The bits can be detected by using a threshold detector by setting the threshold to zero. For any inner product with a positive value we detect bit 0, and for a negative value, we detect bit 1. Thus, for channel one we detect bit 1, and for channel two, we detect bits 0 and 1.

3. Fading Channel

Fading is used to describe the deviation or fluctuations of the amplitudes, phases or multipath delays of a radio signal over a short period of time or travel distance. The presence of multiple paths between the transmitter and receiver introduces complexity in both channel model and receiver. Due to distortion, the received signal often has little resemblance to the transmitted waveform. This is particularly true for transmissions in an urban environment, where propagation is disturbed by a number of interfering objects

such as buildings or trees. The presence of multipath can impose severe limitations on receiver performance. This can be mitigated if a detailed characterization of the multipath channel is available at the receiver.

a. Multipath Channel

Multipath occurs when a transmit signal is reflected by objects such as walls in the environment before reaching the receiver. The reflected signals may arrive with random offsets in time or phase, as illustrated using an impulse response in Figure 9. These effects result in multiple versions of the transmitted signal that arrive and combine at the receiving antenna and can potentially lead to errors on the receiver side as it attempts to correctly detect and decode the distorted received signal. The combined received signal can be modeled as a Rayleigh fading process (for no line-of-sight), a Rician fading process (with one line-of-sight path), or a Nakagami fading process [2].

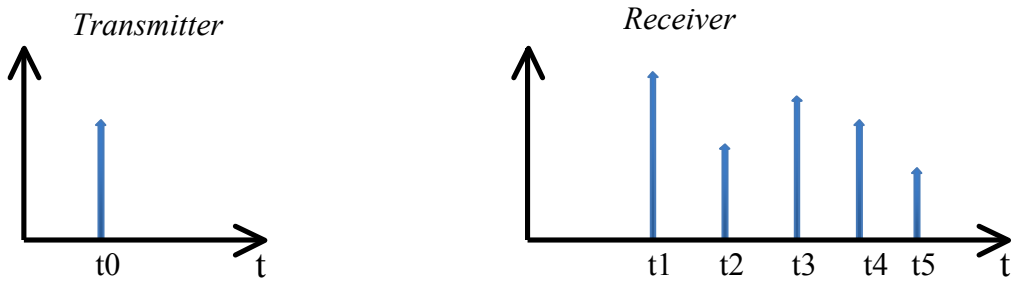

Figure 9. Impulse response of multipath channel is illustrated.

b. Rayleigh Fading

In radio channels, the Rayleigh distribution is commonly used to describe the statistical time-varying nature of the received envelope of a flat fading signal or the envelope of an individual multipath component. The Rayleigh distribution is given by [12]

$$p(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right),\quad (0\le r\le\infty) \tag{2.11}$$

where σ is the rms of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection.

A channel is defined as flat fading when the multipath delay spread is less than the symbol time [2]. For a flat fading, the complex channel tap coefficient is $h = |h| e^{j\theta}$, where h is the attenuation coefficient of the signal, and θ is the phase shift that the fading channel introduces. For this thesis, it is assumed that the complex channel tap is available via perfect channel estimation.

4. Maximal Ratio Combining (MRC)

In an L-fold diversity system, assuming fixed signal bandwidth and transmitted power, the energy of a transmitted copy is $1/L$ the symbol energy. Thus, to detect the received signal, an optimum coherent combiner in a receiver must obtain a combining decision sample whose signal-to-noise ratio is the sum of the L instantaneous signal-to-noise ratios. A MRC block diagram is illustrated in Figure 10. The combiner at the end has to align the phases (co-phase) of each of the L received sample before summing (weighting).

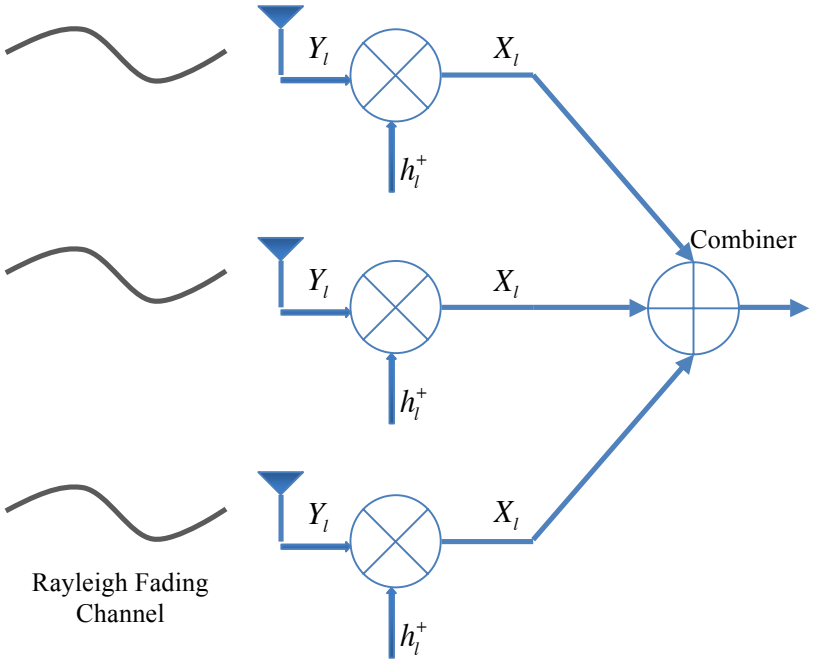

Figure 10. Maximal ratio combining (MRC) diagram is illustrated.

As explained in Chapter 10 of [1], consider the pre-combining samples of L copies of an arbitrary symbol s_i. Each copy has a corresponding complex sub-channel tap of h_i, and the sub-channel received signal Y_i is

$$Y_l = h_l s_l + N_l, \quad l = 1,2,\text{K}\ ,L \tag{2.12}$$

where N_i is the complex AWGN of the *lth* sub-channel with variance $2\sigma^2$. In vector form, the received signal can be written as

$$\boldsymbol{Y} = \boldsymbol{h}s_i + \boldsymbol{N}, \tag{2.13}$$

where $\boldsymbol{Y} = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_L \end{bmatrix}^t$, $\boldsymbol{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_L \end{bmatrix}^t$, and $\boldsymbol{N} = \begin{bmatrix} N_1 & N_2 & \cdots & N_L \end{bmatrix}^t$. Assuming perfect channel estimation, we have the sufficient statistic for coherent demodulation is $\frac{\boldsymbol{h}^*}{\|\boldsymbol{h}\|}\boldsymbol{Y}$, where * denotes conjugate transpose of the vector and $\|\ .\ \|$ is the norm of the vector. Thus, the MRC decision variable is

$$X = \frac{\boldsymbol{h}^\dagger}{\|\boldsymbol{h}\|}\left(\boldsymbol{h}s_i + \boldsymbol{N}\right), \tag{2.14}$$

where $\frac{\boldsymbol{h}^*}{\|\boldsymbol{h}\|}\boldsymbol{N}$ has a variance of $2\sigma^2$. The instantaneous MRC output $\boldsymbol{SNR}_{0,i}$ given the symbol s_i is

$$\boldsymbol{SNR}_{0,i} = \frac{\|\boldsymbol{h}\|^2|s_i|^2}{2\sigma^2} = \sum\nolimits_l \frac{|h_l|^2|s_i|^2}{2\sigma^2} = \sum\nolimits_l \boldsymbol{SNR}_{l,i}, \tag{2.15}$$

where $\boldsymbol{SNR}_{l,i} = \frac{|h_l|^2|s_i|^2}{2\sigma^2}$ is the instantaneous signal-to-noise ratio of the pre-combining l^{th} copy of symbol s_i. Thus, MRC achieves the maximum output signal-to-noise ratio, which is the sum of L input ***SNR***s.

5. MIMO Space-time Coded System

MIMO systems can be defined simply as systems for which the transmitting and the receiving ends are equipped with multiple antenna elements [13]. A MIMO setup is illustrated in Figure 11.

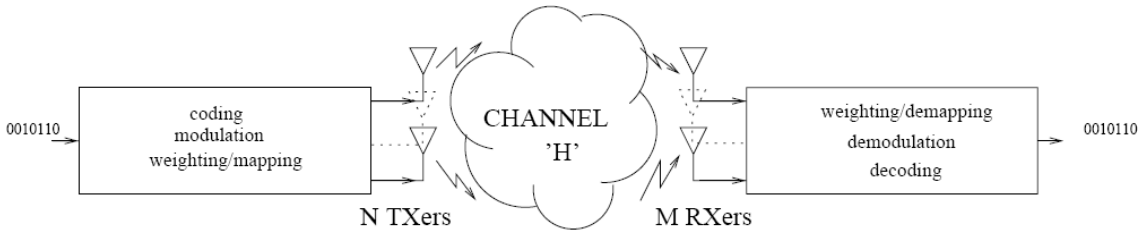

Figure 11. Diagram of a MIMO system is illustrated (From [13]).

The core idea in MIMO systems is that a space-time signal in which the signal time is complemented by the spatial dimension is inherent in the use of multiple spatially distributed antennas. The signals on the transmit antennas at one end and the receive antennas at the other end are designed in such a way that performance is improved. As such, MIMO has the ability to take advantage of multipath fading propagation.

a. Space-time Coding

A typical digital communication system consists of a transmitter and a receiver. Space-time coding involves the use of multiple transmit antennas and receive antennas, as illustrated in Figure 12. Space-time coding is a scheme that achieves a full diversity order by joint encoding of multiple transmit antennas [13]. In this scheme, a number of code symbols equal to the number of transmit antennas are generated and transmitted simultaneously, one symbol from each antenna. These symbols are generated by a space-time encoder so that by using appropriate signal processing and decoding at the receiver, diversity gain can be maximized.

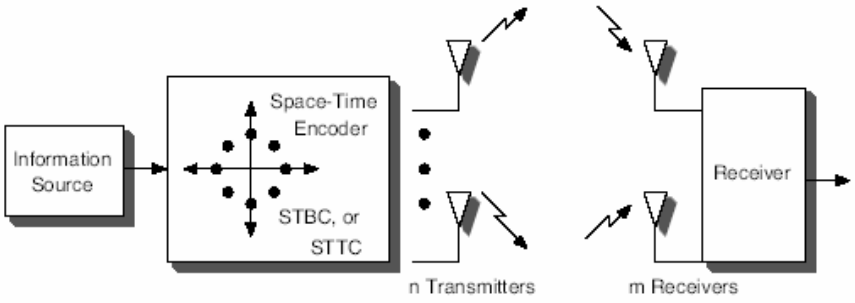

Figure 12. Model of space-time coding is shown (From [13]).

The input symbols from the information source to the space-time encoder are encoded into groups of symbols by space-time block code (STBC) or space-time trellis code (STTC). In a given symbol period, the symbols are transmitted simultaneously from the transmit antennas. In space-time coding, information is transmitted in the form of data sequence. Data are coded and sent along with respect to time. Space-time coding uses the knowledge of the amount of coded data with respect to time and the number of transmit antennas.

Alamouti [4] devised a simple transmitter diversity scheme with orthogonal space-time block coding (OSTBC) for transmission with two antennas as shown in Figure 13.

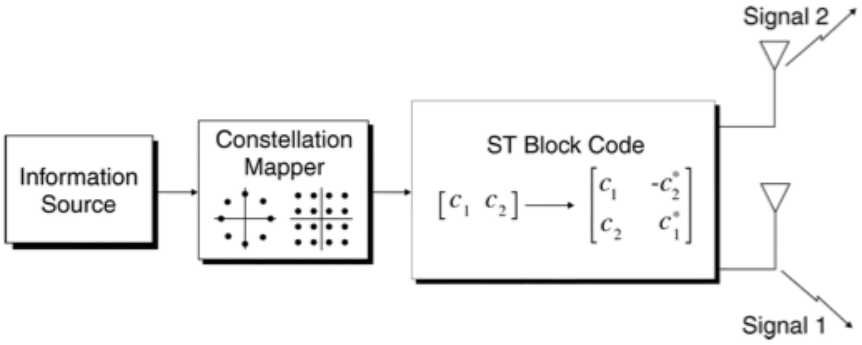
Signal 2
Information Source
Constellation Mapper
ST Block Code
$[c_1 \ c_2] \longrightarrow \begin{bmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{bmatrix}$
Signal 1

Figure 13. Transmitter diversity with space-time block coding is shown (From [13]).

In this scheme, the input symbols to the space-time block encoder are divided into groups of two symbols each. At a given symbol period, the two symbols in

each group $\{C_1, C_2\}$ are transmitted simultaneously from the two antennas. The information is encoded and sent through the antenna with respect to time. The code is specified by the matrix $\mathbf{G}$ whose code words $\begin{bmatrix} C_1 & -C_2^* \end{bmatrix}$ and $\begin{bmatrix} C_2 & C_1^* \end{bmatrix}$ are the columns of $\mathbf{G}$:

$$\mathbf{G} = \begin{bmatrix} C_1 & C_2 \\ -C_2^* & C_1^* \end{bmatrix} \begin{matrix} \xrightarrow{\textit{Antenna}} \\ \\ \Big\downarrow \textit{Symbol time} \end{matrix}$$

** denotes the complex conjugate*

From the first symbol period, the signal transmitted from antenna one is C_1 and the signal transmitted from antenna two is C_2. In the next symbol period, the signal $-C_2^*$ is transmitted from antenna one, and C_1^* is transmitted from antenna two. The columns and rows of matrix $\mathbf{G}$ are orthogonal; that is, $\mathbf{G}^*\mathbf{G} = \mathbf{G}\mathbf{G}^* = \|\mathbf{C}\|^2 \mathbf{I}_{2\times 2}$, where $\mathbf{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}^t$. The Alamouti code allows the transmission of two symbols over two antennas in two consecutive times without the additional expense of bandwidth. The sum of the channel-tap weighted code words plus noise is [1]

$$\begin{aligned} \mathbf{Z} &= h_1 \begin{bmatrix} C_1 \\ -C_2^* \end{bmatrix} + h_2 \begin{bmatrix} C_2 \\ C_1^* \end{bmatrix} + \begin{bmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{bmatrix} \\ &= \begin{bmatrix} h_1 C_1 + h_2 C_2 \\ h_2 C_1^* - h_1 C_2^* \end{bmatrix} + \begin{bmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{bmatrix} = \mathbf{G}\boldsymbol{h} + \boldsymbol{\mathcal{N}}. \end{aligned} \tag{2.16}$$

The vector $\mathbf{Z} = \mathbf{G}\boldsymbol{h} + \boldsymbol{\mathcal{N}}$ can be mapped into $\mathbf{Y} = \mathbf{H}\mathbf{C} + \boldsymbol{N}$ by changing the second element of $\mathbf{Z} = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}^t$, which is $Z_2 = h_2 C_1^* - h_1 C_2^* + \mathcal{N}_2$, into its complex conjugate $Z_2^* = h_2^* C_1 - h_1^* C_2 + \mathcal{N}_2^*$. Thus, **Y** can be expressed as

$$\mathbf{Y} = \mathbf{HC} + \mathbf{N} = \begin{bmatrix} Z_1 \\ Z_2^* \end{bmatrix} = \begin{bmatrix} h_1 C_1 + h_2 C_2 \\ h_2^* C_1 - h_1^* C_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}, \tag{2.17}$$

where $\mathbf{N} = \begin{bmatrix} N_1 & N_2 \end{bmatrix}^t = \begin{bmatrix} \mathcal{N}_1 & \mathcal{N}_2^* \end{bmatrix}^t$, and **H,** the channel tap matrix of the code matrix **G**, is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}. \tag{2.18}$$

b. Orthogonal space-time Block Coding

Further explained in [2], the m symbols $\hat{\mathbf{d}} = \begin{bmatrix} d_1 & d_2 & \cdots & d_m \end{bmatrix}$ to be transmitted are coded by the OSTBC **G** [2], and the received signal is given by

$$\mathbf{Z} = \mathbf{Gh} + N, \tag{2.19}$$

where the complex code **G** is an $L_s \times L_t$ matrix, where L_t is the number of transmit antennas for m complex symbols transmitted over L_s symbols times. Thus, the code rate is $R = m / L_s$. The parameter N is the $L_s \times 1$ AWGN vector at the receiver with covariance matrix $\sigma^2 \mathbf{I}_{L_s \times L_s}$, and **h** is the $L_t \times 1$ complex channel attenuation vector where $\mathbf{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_{L_t} \end{bmatrix}^t$. From (2.16) and (2.17), the code **G** is designed such that the received signal can be transformed to a convenient form

$$\widehat{\mathbf{Y}} = \mathbf{H}\hat{\mathbf{d}} + \widehat{\mathbf{N}}, \tag{2.20}$$

where $\hat{\mathbf{d}}$ is the symbol sequence defined previously, and the $L_t \times 1$ $\widehat{\mathbf{N}}$ is the preserved noise vector. The variance is preserved, and the covariance of the noise vector is given by

$\sigma^2 \mathbf{I}_{L_t \times L_t}$. The $L_t \times m$ matrix **H** is termed as the channel tap matrix whose row entries are mainly derived from **h** such that the columns of **H** are orthogonal to each other; i.e., $\| \mathbf{h} \|^2 \mathbf{I}_{m \times m} = \mathbf{H}^{+} \mathbf{H}$ or, equivalently, $\| \mathbf{h} \|^2 \mathbf{I}_{m \times m} = \mathbf{H}^{t} \mathbf{H}^{*}$, where $\| \mathbf{h} \|$ is clearly the norm of **h**.

C. TYPES OF JAMMING INTERFERENCE

In general, jamming in digital communication means the prevention of successful radio communications by the use of electromagnetic signals. The aim of communications jamming is to cause interference to a system and cause the system's performance to degrade. In this thesis, three types of jamming, namely, barrage jamming, pulse-noise jamming and tone jamming, are covered.

1. Barrage Jamming [1]

Barrage jamming is jamming that is due to barrage noise, which can be modeled as broadband noise, i.e., additive white gaussian noise (AWGN). The channel is modeled as an AWGN channel with the presence of a broadband jamming signal whose power spectral density (PSD) is flat and is denoted as $J_0 / 2$.

2. Pulsed-Noise Jamming [1] [3]

Pulsed-noise jamming is an effective jamming strategy against a direct sequence spread spectrum signal. It is jamming caused by broadband noise received in periodic spurts. The period or duty cycle is given by ρ, which is between the value of 0 to 1. For the special case where $\rho = 0$, the jammer is specified to be turned off, and the receiver interference is just due to the receiver noise. By the total probability theorem, the BER is given by [3]

$$P_b = \rho P_{bJ+N} + (1 - \rho) P_{bN}, \tag{2.21}$$

where ρP_{bJ+N} is the BER with jamming and receiver noise and P_{bN} is the BER with AWGN only.

3. Tone Jamming [2]

In tone jamming, the jamming waveform is presumably within the signal bandwidth. An interference tone at the carrier frequency is sent to jam that carrier frequency. This causes a phase offset after the received signal is downconverted from passband to baseband. The jamming signal is given by

$$j(t) = A_J \cos\left(2\pi f_c t + \theta_J\right), \tag{2.22}$$

where A_J is the amplitude and θ_J is the arbitrary phase of the jamming signal. In direct sequence spread spectrum, the variances due to the jamming tone on the I and Q channels are equal and are given by [2]

$$\sigma_{J_I}^2 = \sigma_{J_Q}^2 = \frac{A_J^2 T_c}{8N}, \tag{2.23}$$

where T_c is the spreading chip time and N is the spreading gain; thus, the total interference variance due to jammer and noise at the detector is

$$\sigma_T^2 = \frac{A_J^2 T_c}{4N}. \tag{2.24}$$

D. SUMMARY

The signal model background and the various techniques used in this thesis were provided in this chapter. Each technique has its own advantages in improving the system performance. Spread spectrum is used to improve the jamming capability for the system. Orthogonal covering, which employs Walsh function, is used to provide orthogonal capability and channeling for both fixed and multi-rate data transmission. MRC improves the received SNR of a system in fading channel by adding all multipath signal SNRs. Spatial diversity via MIMO technique is used to combat fading channel and improves the system performance. The types of jamming interferences were also briefly described.

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III. COMPLEX SPREAD SPECTRUM SIGNALING WITH ORTHOGONAL COVERING IN FIXED DATA RATE APPLICATIONS

A. INTRODUCTION

The orthogonal covering codes are assigned to multi-rate user signals to provide orthogonalilty so that the signals can be multiplexed into one single carrier without interfering with one another. When the spreading sequence is used, the overall bandwidth of the signal is increased. A spread spectrum signal has a much larger bandwidth due to the spreading in the frequency domain. A direct sequence spread spectrum signal with spreading factor *N* can reduce the jamming power at the receiver via de-spreading. It is noted that the direct sequence code shift keying (DS-CSK) technique is employed in the reverse link of IS-95. This scheme is not practical for our system model, as explained in approach one. A scheme that can fit our design is devised, as explained in approach two, and is followed by the derivation of the procedure in Section B.

Approach one: Let $k = \log_2 \mathbf{M}$, where *k* is the number of bits per symbol for **M**-ary modulation and **M** is the number of bits. The symbols are assigned a distinct Walsh sequence of *M* chips from the set of *M* Walsh sequences obtained from the code tree. Assume a bit sequence of -1s and 1s, and R_b is the bit rate. The symbol rate $R_s = 1/T_s$ is R_b / k, where T_s is the symbol period. Also, assume a Walsh sequence of *M* chips (1s and -1s) with chip time of T_w, where $T_s = MT_w$. To employ Walsh coding, each data symbol is multiplied by the Walsh sequence to form a Walsh-data vector, where the Walsh chip rate is $R_w = MR_s$. Consequently, to perform the direct sequence spreading, a periodic PN sequence of *N* chips (1s and -1s) is considered with chip time of $T_c = 1/R_c$, where R_c is the chip rate of the PN sequence. Then, R_c is selected to be an integer multiple of the Walsh chip rate R_w; that is, $R_c = NR_w$. The Walsh-data vector is then multiplied by the PN sequence to form the Walsh coding and spreading signal, and the final chip rate is $R_c = NMR_s$. Hence, the overall spreading factor is *NM*, which also

means the spread signal bandwidth is *NM* times larger then the data symbol rate. This procedure is illustrated in Figure 14.

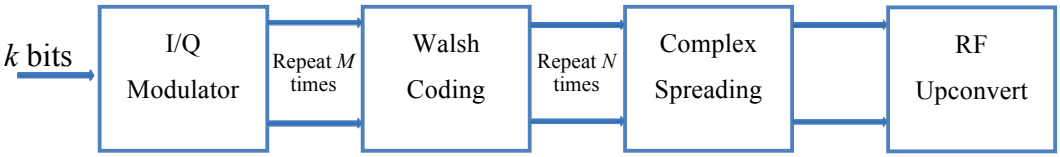

Figure 14. Walsh coding and complex spreading procedure is shown.

This method is not practical due to the large bandwidth usage. With a 64 chip-Walsh sequence and a 128 chip-PN sequence, the spreading factor is $64 \times 128 = 8192$, and the bandwidth is 8192 times larger. The use of short PN codes is not suitable, as the protection against jamming is reduced.

Approach two: A scheme is devised which integrates the Walsh coding with the spreading signal. This method can achieve the desired bandwidth expansion factor of *N*. It is also easier to implement. Firstly, each symbol is spread with the PN sequence to form the spreading sequence. The PN chip rate is $R_c = NR_s$. In practice, the spread factor *N* is usually larger than *M* and *N* is chosen such that $N = bM$ where *b* is a positive integer and a power of two. Before employing the Walsh coding, the Walsh vector of spread factor *M* has to be repeated *b* times so that it has the same chip rate as that of the PN sequence; i.e., $R_w = R_c$. Walsh coding is applied by multiplying each of the repeated Walsh codes with each of the spread symbol to form the integrated spreading sequence. It can be seen that the overall chip rate is $R_c = NR_s$. Thus, the spread bandwidth only depends on *N*. This method can allow us to use long PN codes and, at the same time, provide orthogonality capability without increasing the overall bandwidth. The procedure is illustrated in Figure 15.

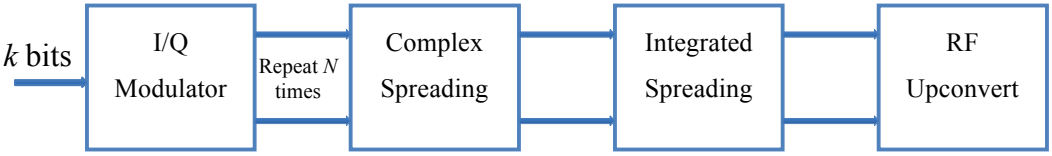

Figure 15. Integrated spreading procedure 1 is illustrated.

Another method is to apply Walsh coding to the symbol before spreading. Each symbol is repeated M times and coded with the Walsh vector. The chip rate is $R_w = MR_s$. Then, each Walsh-data vector is repeated b times to obtain the same chip rate as the PN sequence. To achieve the integrated spreading, we multiply the corresponding elements of the repeated Walsh-data vector and the PN code. Thus, the chip rate is $R_c = bR_w = bMR_s = NR_s$. The procedure is illustrated in Figure 16.

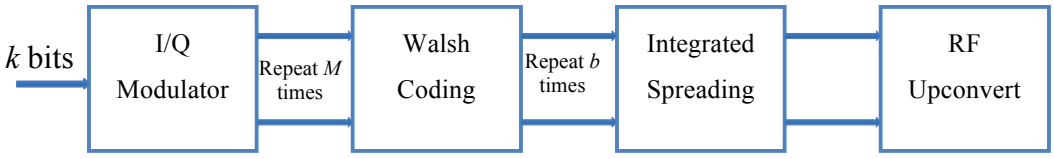

Figure 16. Integrated spreading procedure 2 is illustrated.

The repeated Walsh sequence procedure can be carried out in two methods. The first method is to repeat each Walsh chip in the sequence. For example, let us take the Walsh vector $\boldsymbol{w} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}$, and each of the Walsh chip is repeated two times i.e., $b = 2$. The repeated Walsh vector is $\tilde{\boldsymbol{w}} = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}$. From the code tree (in Figure 8), it can be seen that this repeated sequence is the successor of the other branch sequence; thus, it is not suitable for use in multi-rate user transmission. This is because it will affect the availability of the remaining Walsh code resources, i.e., the system capacity. The advantage of this method is it can provide better burst error performance in fixed rate transmission.

The second method is to repeat the whole Walsh sequence. Taking the same example as above, the repeated Walsh vector is $\tilde{\boldsymbol{w}} = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$. Notice the repeated vector is the successor of the Walsh sequence, i.e., the next branch in the code tree. Therefore, it does not affect the orthogonality of the sequence and the availability of the Walsh code resources. This method is more suitable for multi-rate users transmission and can also work in conjunction with other Walsh code assignment algorithms.

B. INTEGRATED COMPLEX SPREADING WITH WALSH CODING PROCEDURE

Here, the procedures to illustrate the use of orthogonal covering of the spread spectrum signals for multiuser application in the section C and multi-rate users application in Chapter IV are discussed. The procedures for each section are explained mathematically and followed by an example.

1. Complex Spreading Procedure

Let **D** be a $1\times m$ row vector of m symbols to be transmitted from one channel, where $D_n = d_n + j\hat{d}_n$ is an arbitrary complex symbol from **D**. The d is the I-value, and $\hat{d}$ is the Q-value. Let a complex PN vector be $\boldsymbol{c} = \boldsymbol{c}_I + j\boldsymbol{c}_Q$ with a spreading factor of N. The symbols in **D** can be spread with the same PN code or each symbol in **D** can be spread with a different code. Assume all symbols in **D** are spread with the same PN code; then spread matrix **S** is given by

$$\mathbf{S} = \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D}, \tag{3.1}$$

where $\|\boldsymbol{c}\|$ is used as a normalization to preserve the symbol energies in **D**. The $N\times m$ spread matrix **S** and is given by

$$\mathbf{S} = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}\begin{bmatrix} D_1 & D_2 & \cdots & D_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_1 & \boldsymbol{s}_2 & \cdots & \boldsymbol{s}_m \end{bmatrix}. \tag{3.2}$$

The nth column is the $N\times 1$ spread vector given by $\boldsymbol{s}_n = D_n\frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}$ and the complex spread vector $\boldsymbol{s}_n$ takes the form $\boldsymbol{s}_n = \boldsymbol{s}_{nI} + j\boldsymbol{s}_{nQ}$ where

$$\begin{aligned} \boldsymbol{s}_n = D_n \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|} &= \frac{1}{\|\boldsymbol{c}\|}\left(d_n + j\hat{d}_n\right)\left(\boldsymbol{c}_I + j\boldsymbol{c}_Q\right) \\ &= \frac{1}{\|\boldsymbol{c}\|}\left(d_n\boldsymbol{c}_I - \hat{d}_n\boldsymbol{c}_Q\right) + j\frac{1}{\|\boldsymbol{c}\|}\left(\hat{d}_n\boldsymbol{c}_I + d_n\boldsymbol{c}_Q\right), \end{aligned} \tag{3.3}$$

and the I and Q spread vectors are given by

$$\boldsymbol{s}_{nI} = \frac{1}{\|\boldsymbol{c}\|}\left(d_n\boldsymbol{c}_I - \hat{d}_n\boldsymbol{c}_Q\right), \qquad \boldsymbol{s}_{nQ} = \frac{1}{\|\boldsymbol{c}\|}\left(\hat{d}_n\boldsymbol{c}_I + d_n\boldsymbol{c}_Q\right). \tag{3.4}$$

2. Walsh Coding Procedure

Let $\boldsymbol{w}$ be the $1\times M$ Walsh vector of spreading factor M which is given by $\boldsymbol{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}$. As mentioned before, the PN spreading factor N is the integer multiple of the Walsh spreading factor M, where $N = bM$. Thus, the Walsh vector has to be repeated b times to match the PN sequence chip rate. To integrate the repeated Walsh vector with the spread matrix **S**, we define the $N\times N$ diagonal Walsh matrix $\mathbf{W}$ such that each Walsh chip can be multiplied with each spread chip. The diagonal Walsh matrix $\mathbf{W}$ can be obtained by two methods as follows:

*Method one: Each Walsh chip in **w** is repeated b times*

$$\mathbf{W} = diagonal\left[\begin{array}{cccc} \underbrace{w_1\cdots w_1}_{b}, & \underbrace{w_2\cdots w_2}_{b}, & \cdots, & \underbrace{w_M\cdots w_M}_{b} \end{array}\right], \tag{3.5}$$

*Method two: The Walsh vector **w** is repeated b times*

$$\mathbf{W} = diagonal\left[\begin{array}{cccc} \underbrace{\begin{bmatrix} w_1 & w_2\cdots w_M \end{bmatrix}}_{1}, & \cdots, & \underbrace{\begin{bmatrix} w_1 & w_2\cdots w_M \end{bmatrix}}_{b} \end{array}\right], \tag{3.6}$$

where the "diagonal" operation means to create a matrix where the diagonal elements are made up of the repeated Walsh vector elements. With either method, Walsh coding is achieved by multiplying the spread matrix **S** with **W** to form the integrated spread matrix given by

$$\mathbf{X} = \mathbf{WS} = \begin{bmatrix} \boldsymbol{x}_1 & \boldsymbol{x}_2 & \cdots & \boldsymbol{x}_m \end{bmatrix}, \tag{3.7}$$

where the integrated spread matrix **X** is a $N \times m$ matrix, and the $N \times 1$ column vector $\boldsymbol{x}_n$ is the integrated spread vector which consists of *b* repeated Walsh codes or Walsh vectors depending on the method used. The complex integrated spread vector $\boldsymbol{x}_n$ takes the form $\boldsymbol{x}_n = \boldsymbol{x}_{nI} + j\boldsymbol{x}_{nQ}$, and (3.4) is modified to

$$\begin{aligned} \boldsymbol{x}_n = \mathbf{W}\boldsymbol{s}_n &= \mathbf{W}D_n \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|} \\ &= \frac{\mathbf{W}}{\|\boldsymbol{c}\|}\left(d_n + j\hat{d}_n\right)\left(\boldsymbol{c}_I + j\boldsymbol{c}_Q\right) \\ &= \frac{\mathbf{W}}{\|\boldsymbol{c}\|}\left(d_n\boldsymbol{c}_I - \hat{d}_n\boldsymbol{c}_Q\right) + j\frac{\mathbf{W}}{\|\boldsymbol{c}\|}\left(\hat{d}_n\boldsymbol{c}_I + d_n\boldsymbol{c}_Q\right), \end{aligned} \tag{3.8}$$

where the I and Q integrated spread vectors are given by

$$\begin{aligned} \boldsymbol{x}_{nI} &= \frac{\mathbf{W}}{\|\boldsymbol{c}\|}\left(d_n\boldsymbol{c}_I - \hat{d}_n\boldsymbol{c}_Q\right) \\ \boldsymbol{x}_{nQ} &= \frac{\mathbf{W}}{\|\boldsymbol{c}\|}\left(\hat{d}_n\boldsymbol{c}_I + d_n\boldsymbol{c}_Q\right). \end{aligned} \tag{3.9}$$

3. Complex De-spreading Procedure

The goal of the de-spreading operation is to recover the Walsh vectors and is achieved by multiplying each element in the columns of **X** by the complex conjugate of PN vector ***c***. Thus, to de-spread the received integrated spread matrix **X**, we define the $N \times N$ diagonal PN matrix ***C*** given by

$$\boldsymbol{C} = diagonal\begin{bmatrix} c_1 & c_2 & \cdots & c_N \end{bmatrix}. \tag{3.10}$$

Hence, the de-spread $N \times m$ repeated Walsh matrix is expressed in the form

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{W}\mathbf{S} = \frac{\mathbf{W}\boldsymbol{C}^*\boldsymbol{c}\mathbf{D}}{\|\boldsymbol{c}\|^2}, \tag{3.11}$$

where $[\bullet]^*$ denotes the complex conjugate.

4. Symbol Recovery Procedure

Now, we proceed to the symbol recovery procedure to recover the original symbol vector **D**. Firstly, we define the $1\times N$ repeated Walsh vector $\widetilde{\boldsymbol{w}}$. Again, these are two methods as follows:

Method one:

$$\widetilde{\boldsymbol{w}}=\left[\ \underbrace{w_1\cdots w_1}_{b},\quad \underbrace{w_2\cdots w_2}_{b},\quad \cdots,\quad \underbrace{w_M\cdots w_M}_{b}\ \right], \tag{3.12}$$

where $\widetilde{\boldsymbol{w}}$ is the diagonal of the $N\times N$ matrix **W** in (3.5).

Method two:

$$\widetilde{\boldsymbol{w}}=\left[\ \underbrace{\left[w_1\ w_2\cdots w_M\right]}_{1},\quad \cdots,\quad \underbrace{\left[w_1\ w_2\cdots w_M\right]}_{b}\ \right], \tag{3.13}$$

where $\widetilde{\boldsymbol{w}}$ is the diagonal of the $N\times N$ matrix **W** in (3.6). The symbol recovery procedure is achieved by multiplying the repeated Walsh vector $\widetilde{\boldsymbol{w}}$ and **V**; hence, the symbol vector **D** can be recovered via

$$\mathbf{Z}=\widetilde{\boldsymbol{w}}\mathbf{V}=\frac{\widetilde{\boldsymbol{w}}\mathbf{W}\boldsymbol{C}^{*}\boldsymbol{c}\mathbf{D}}{\|\boldsymbol{c}\|^{2}}. \tag{3.14}$$

With *method one* the recovered symbol vector is

$$\begin{aligned}\mathbf{Z}&=\frac{1}{\|\boldsymbol{c}\|^{2}}\underbrace{\left[\ w_1\cdots w_1,\ \cdots,\ w_M\cdots w_M\ \right]}_{1\times N}\underbrace{\begin{bmatrix} w_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \ddots & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & w_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \ddots & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & w_M & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & w_M \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1^{*} & 0 & 0 & 0\\ 0 & c_2^{*} & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & c_N^{*} \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1\\ c_2\\ \vdots\\ c_N \end{bmatrix}}_{N\times 1}\mathbf{D}\\ &=\frac{1}{\|\boldsymbol{c}\|^{2}}\underbrace{\left[\ w_1^{2}\cdots w_1^{2},\ \cdots,\ w_M^{2}\cdots w_M^{2}\ \right]}_{1\times N}\underbrace{\begin{bmatrix} |c_1|^{2}\\ |c_2|^{2}\\ \vdots\\ |c_N|^{2} \end{bmatrix}}_{N\times 1}\mathbf{D}\\ &=\frac{1}{\|\boldsymbol{c}\|^{2}}\left\{w_1^{2}\left(|c_1|^{2}+\cdots+|c_b|^{2}\right)+w_2^{2}\left(|c_{b+1}|^{2}+\cdots+|c_{2b}|^{2}\right)+\cdots+w_M^{2}\left(|c_{N-b+1}|^{2}+\cdots+|c_N|^{2}\right)\right\}\mathbf{D}.\end{aligned} \tag{3.15}$$

Since $w_1^2 = w_2^2 = \cdots = w_M^2 = 1$, we have

$$\mathbf{Z} = \frac{1}{\|\boldsymbol{c}\|^2}\left(|c_1|^2 + |c_2|^2 + \cdots + |c_N|^2\right)\mathbf{D} = \frac{\|\boldsymbol{c}\|^2}{\|\boldsymbol{c}\|^2}\mathbf{D} = \mathbf{D}. \tag{3.16}$$

With *method two* for the repeated Walsh vector, we have

$$\begin{aligned}\mathbf{Z} &= \frac{1}{\|\boldsymbol{c}\|^2}\underbrace{\left[\left[w_1 \cdots w_M\right], \cdots, \left[w_1 \cdots w_M\right]\right]}_{1\times N}\underbrace{\begin{bmatrix} w_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_M & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_M \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}}_{N\times 1}\mathbf{D} \\ &= \frac{1}{\|\boldsymbol{c}\|^2}\underbrace{\left[\left[w_1^2\ w_2^2 \cdots w_M^2\right], \cdots, \left[w_1^2\ w_2^2 \cdots w_M^2\right]\right]}_{1\times N}\underbrace{\begin{bmatrix} |c_1|^2 \\ |c_2|^2 \\ \vdots \\ |c_N|^2 \end{bmatrix}}_{N\times 1}\mathbf{D} \\ &= \frac{1}{\|\boldsymbol{c}\|^2}\left(w_1^2|c_1|^2 + w_2^2|c_2|^2 + \cdots w_M^2|c_M|^2 + \cdots + w_1^2|c_{N-M+1}|^2 + w_2^2|c_{N-M+2}|^2 + \cdots w_M^2|c_N|^2\right)\mathbf{D}.\end{aligned} \tag{3.17}$$

Since $w_1^2 = w_2^2 = \cdots = w_M^2 = 1$, we have

$$\mathbf{Z} = \frac{1}{\|\boldsymbol{c}\|^2}\left(|c_1|^2 + |c_2|^2 + \cdots + |c_N|^2\right)\mathbf{D} = \frac{\|\boldsymbol{c}\|^2}{\|\boldsymbol{c}\|^2}\mathbf{D} = \mathbf{D}. \tag{3.18}$$

5. Example 1

The integrated spreading and the use of repeated Walsh vector in methods one and two are illustrated. Assuming QPSK signals, we consider a row vector of two complex symbols to be transmitted $\mathbf{D} = \begin{bmatrix} 1+j & 1-j \end{bmatrix}$. Also we assume that the symbols in **D** are spread with the same complex PN vector given by $\boldsymbol{c} = \begin{bmatrix} 1+j & -1-j & -1-j & -1-j & 1+j & -1-j & 1+j & 1+j \end{bmatrix}^t$ and the Walsh sequence is $\boldsymbol{w} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^t$. The spreading factor of PN code is $N = 8$, and the spreading factor of the Walsh sequence is $M = 4$. The complex spreading matrix **S** is given by

$$\mathbf{S} = \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D} = \underbrace{\frac{1}{4}}_{\frac{1}{\|\boldsymbol{c}\|}} \underbrace{\begin{bmatrix} 1+j \\ -1-j \\ -1-j \\ -1-j \\ 1+j \\ -1-j \\ 1+j \\ 1+j \end{bmatrix}}_{N\times 1} \underbrace{\begin{bmatrix} 1+j & 1-j \end{bmatrix}}_{1\times m} = \frac{1}{4}\begin{bmatrix} 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \\ 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \end{bmatrix}. \tag{3.19}$$

To perform the integrated spreading, we define the 8×8 diagonal Walsh matrix $\mathbf{W}$ such that each Walsh chip can be multiplied with each spread chip. Given that the PN spreading factor is $N = 8$ and the Walsh sequence spreading factor is $M = 4$, the repeated diagonal Walsh vector is given by two methods:

Method one:

$$\mathbf{W} = diagonal\begin{bmatrix} 1 & \underline{1} & -1 & \underline{-1} & 1 & \underline{1} & -1 & \underline{-1} \end{bmatrix}, \tag{3.20}$$

Method two:

$$\mathbf{W} = diagonal\begin{bmatrix} 1 & -1 & 1 & -1 & \underline{1} & \underline{-1} & \underline{1} & \underline{-1} \end{bmatrix}, \tag{3.21}$$

where the underscore denotes the repeated chip. For *method one*, the Walsh coding is obtained by multiplying the diagonal Walsh matrix $\mathbf{W}$ with the spread matrix $\mathbf{S}$. We have

$$\mathbf{X} = \mathbf{WS}$$

$$= \underbrace{\frac{1}{4}}_{\frac{1}{\|\boldsymbol{c}\|}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{-1} \end{bmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \\ 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \end{bmatrix}}_{\boldsymbol{c}\mathbf{D}} = \frac{1}{4}\begin{bmatrix} 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \end{bmatrix}. \tag{3.22}$$

To perform de-spreading, each element in the columns of **X** is multiplied by the complex conjugate of PN vector ***c***. The 8×8 diagonal PN matrix ***C*** is defined in (3.10), and the de-spread "repeated" Walsh matrix is given by

$$\mathbf{V}=\frac{\boldsymbol{C}^{*}}{\|\boldsymbol{c}\|}\mathbf{X}=\underbrace{\frac{1}{16}}_{\frac{1}{\|\boldsymbol{c}\|^{2}}}\underbrace{\begin{bmatrix} 1-j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1+j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-j \end{bmatrix}}_{\boldsymbol{C}^{*}}\underbrace{\begin{bmatrix} 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \end{bmatrix}}_{\mathbf{WcD}} \quad (3.23)$$

$$=\frac{1}{16}\begin{bmatrix} 2+2j & 2-2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ -2-2j & -2+2j \end{bmatrix}.$$

Before proceeding to the symbol recovery procedure, we define the $1\times N$ repeated Walsh vector $\widetilde{\boldsymbol{w}}$, which is also the diagonal of the $N\times N$ matrix **W** in (3.20) and is given by

$$\widetilde{\boldsymbol{w}}=\begin{bmatrix} 1 & \underline{1} & -1 & \underline{-1} & 1 & \underline{1} & -1 & \underline{-1} \end{bmatrix}. \quad (3.24)$$

The repeated Walsh vector $\widetilde{\boldsymbol{w}}$ is multiplied by the de-spread repeated Walsh vector **V** to obtain the recovered symbols. Thus, we have

$$\mathbf{Z} = \widetilde{\boldsymbol{w}}\mathbf{V} = \underbrace{\frac{1}{16}}_{\frac{1}{\|\boldsymbol{c}\|^2}}\underbrace{\begin{bmatrix} 1 & \underline{1} & -1 & -\underline{1} & 1 & \underline{1} & -1 & -\underline{1} \end{bmatrix}}_{\widetilde{\boldsymbol{w}}}\underbrace{\begin{bmatrix} 2+2j & 2-2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ -2-2j & -2+2j \end{bmatrix}}_{\mathbf{W}\boldsymbol{C}^{*}\boldsymbol{c}\mathbf{D}} \tag{3.25}$$

$$= \frac{1}{16}\begin{bmatrix} 16+16j & 16-16j \end{bmatrix} = \begin{bmatrix} 1+j & 1-j \end{bmatrix} = \mathbf{D}.$$

With *method two*, the integrated spreading is achieved by multiplying the diagonal Walsh matrix **W** with the spread matrix **S**. Thus,

$$\mathbf{X} = \mathbf{WS}$$

$$= \underbrace{\frac{1}{4}}_{\frac{1}{\|\boldsymbol{c}\|}}\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}}_{\mathbf{W}}\underbrace{\begin{bmatrix} 2j & 2 \\ -2j & -2 \\ -2j & -2 \\ -2j & -2 \\ 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \end{bmatrix}}_{\boldsymbol{c}\mathbf{D}} = \frac{1}{4}\begin{bmatrix} 2j & 2 \\ 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ -2j & -2 \end{bmatrix}. \tag{3.26}$$

The de-spread repeated Walsh matrix is obtained by multiplying the diagonal PN matrix ***C*** with **X** and is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^{*}}{\|\boldsymbol{c}\|}\mathbf{X} = \underbrace{\frac{1}{16}}_{\frac{1}{\|\boldsymbol{c}\|^2}}\underbrace{\begin{bmatrix} 1-j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1+j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-j \end{bmatrix}}_{\boldsymbol{c}^{*}}\underbrace{\begin{bmatrix} 2j & 2 \\ 2j & 2 \\ -2j & -2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ 2j & 2 \\ -2j & -2 \end{bmatrix}}_{\mathbf{W}\boldsymbol{c}\mathbf{D}} = \frac{1}{16}\begin{bmatrix} 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \end{bmatrix}. \tag{3.27}$$

The $1 \times N$ repeated Walsh vector $\widetilde{\boldsymbol{w}}$ in this case is given by

$$\tilde{\boldsymbol{w}} = \begin{bmatrix} 1 & -1 & 1 & -1 & \underline{1} & \underline{-1} & \underline{1} & \underline{-1} \end{bmatrix}. \tag{3.28}$$

Note that $\tilde{\boldsymbol{w}}$ is also the diagonal of the $N \times N$ matrix $\mathbf{W}$ in (3.21). The recovered symbols is given by

$$\begin{aligned} \mathbf{Z} = \tilde{\boldsymbol{w}}\mathbf{V} &= \frac{1}{16}\begin{bmatrix} 1 & -1 & 1 & -1 & \underline{1} & \underline{-1} & \underline{1} & \underline{-1} \end{bmatrix} \begin{bmatrix} 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \\ 2+2j & 2-2j \\ -2-2j & -2+2j \end{bmatrix} \\ &= \frac{1}{16}\begin{bmatrix} 16+16j & 16-16j \end{bmatrix} = \begin{bmatrix} 1+j & 1-j \end{bmatrix} = \mathbf{D}. \end{aligned} \tag{3.29}$$

Thus, we show that the integrated spreading and recovery of the complex symbols can be achieved using the repeated Walsh methods one and two.

C. INTEGRATED COMPLEX SPREADING WITH ORTHOGONAL COVERING PROCEDURE FOR MULTIUSER CHANNELS

In this section, the procedure for multiuser channels is discussed. Each user channel is now assigned a unique orthogonal Walsh code. There are *K* user channels assigned with *K* unique Walsh codes of the same spreading factor. The procedure for multiuser channels is illustrated in Figure 17

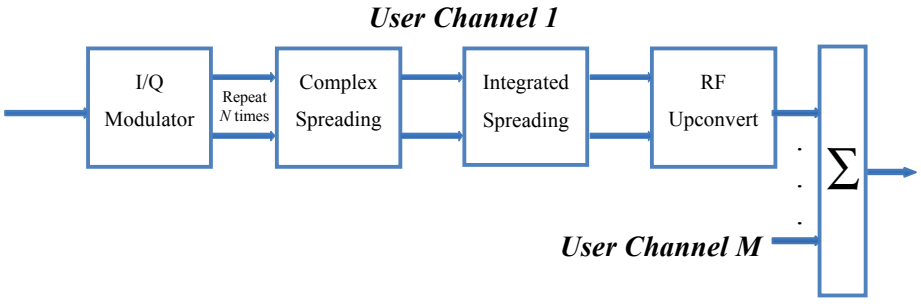

Figure 17. Integrated spreading with orthogonal covering procedure for multiuser is illustrated.

1. Complex Spreading Procedure

Let $\mathbf{D}^{(1)},\mathbf{D}^{(2)},\ldots,\mathbf{D}^{(K)}$ be the data sequences of m complex symbols to be transmitted from K users, where $\mathbf{D}^{(1)}$ is from channel one and $\mathbf{D}^{(K)}$ is from channel K. Let the complex PN $N\times 1$ vector be $\boldsymbol{c}=\boldsymbol{c}_I+j\boldsymbol{c}_Q$. The resulting spread matrix in each channel is given by

$$\mathbf{S}^{(i)}=\frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D}^{(i)}=\left[\begin{array}{cccc}\boldsymbol{s}_1^{(i)} & \boldsymbol{s}_2^{(i)} & \cdots & \boldsymbol{s}_m^{(i)}\end{array}\right], \qquad i=1,2,\ldots,K \tag{3.30}$$

where the *nth* column is the $N\times 1$ spread vector is given by

$$\boldsymbol{s}_n^{(i)}=D_n^{(i)}\frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}=\frac{1}{\|\boldsymbol{c}\|}\left(d_n^{(i)}\boldsymbol{c}_I-\hat{d}_n^{(i)}\boldsymbol{c}_Q\right)+j\frac{1}{\|\boldsymbol{c}\|}\left(\hat{d}_n^{(i)}\boldsymbol{c}_I+d_n^{(i)}\boldsymbol{c}_Q\right), \tag{3.31}$$

where $\boldsymbol{s}_{nI}^{(i)}=\frac{1}{\|\boldsymbol{c}\|}\left(d_n^{(i)}\boldsymbol{c}_I-\hat{d}_n^{(i)}\boldsymbol{c}_Q\right)$ is the I spread vector and $\boldsymbol{s}_{nQ}^{(i)}=\frac{1}{\|\boldsymbol{c}\|}\left(\hat{d}_n^{(i)}\boldsymbol{c}_I+d_n^{(i)}\boldsymbol{c}_Q\right)$ is the Q spread vector.

2. Walsh Coding Procedure

Let the $1\times M$ Walsh vectors be $\boldsymbol{w}^{(1)},\boldsymbol{w}^{(2)},\ldots,\boldsymbol{w}^{(K)}$ where $\boldsymbol{w}^{(1)}$ is assigned to channel one and $\boldsymbol{w}^{(K)}$ is assigned to channel K. Similar to the procedure in Section B.2, the Walsh chip rate is designed to be the same as the PN chip rate. Each Walsh vector $\boldsymbol{w}^{(i)}$ is repeated b times thereby creating the $N\times N$ diagonal Walsh matrix $\mathbf{W}$. Thus, using *method one* it is given by

$$\mathbf{W}^{(i)}=diagonal\left[\begin{array}{cccc}\underbrace{w_1^{(i)}\cdots w_1^{(i)}}_{b}, & \underbrace{w_2^{(i)}\cdots w_2^{(i)}}_{b}, & \cdots, & \underbrace{w_M^{(i)}\cdots w_M^{(i)}}_{b}\end{array}\right] \qquad i=1,2,\ldots,K \tag{3.32}$$

and $\mathbf{W}^{(i)}$ for *method two* is

$$\mathbf{W}^{(i)}=diagonal\left[\begin{array}{ccc}\underbrace{\left[w_1^{(i)}\ w_2^{(i)}\cdots w_M^{(i)}\right]}_{1}, & \cdots, & \underbrace{\left[w_1^{(i)}\ w_2^{(i)}\cdots w_M^{(i)}\right]}_{b}\end{array}\right]. \tag{3.33}$$

To proceed with Walsh coding, the respective *ith* diagonal element of the Walsh matrix $\mathbf{W}^{(i)}$ is multiplied with the *ith* channel spread matrix $\mathbf{S}^{(i)}$, where

$$\mathbf{X}^{(i)} = \mathbf{W}^{(i)}\mathbf{S}^{(i)} = \begin{bmatrix} \boldsymbol{x}_1^{(i)} & \boldsymbol{x}_2^{(i)} & \cdots & \boldsymbol{x}_m^{(i)} \end{bmatrix}. \qquad i = 1,2,\ldots,K \tag{3.34}$$

The integrated spread matrix $\mathbf{X}^{(i)}$ is an $N \times m$ matrix and $\boldsymbol{x}_n^{(i)}$ is the $N \times 1$ integrated spread vector for the respective *ith* channel. The complex integrated spread vector $\boldsymbol{x}_n^{(i)}$ is given by

$$\boldsymbol{x}_n^{(i)} = \mathbf{W}^{(i)}\boldsymbol{s}_n^{(i)} = \frac{\mathbf{W}^{(i)}}{\|\boldsymbol{c}\|}\left(d_n^{(i)}\boldsymbol{c}_I - \hat{d}_n^{(i)}\boldsymbol{c}_Q\right) + j\frac{\mathbf{W}^{(i)}}{\|\boldsymbol{c}\|}\left(\hat{d}_n^{(i)}\boldsymbol{c}_I + d_n^{(i)}\boldsymbol{c}_Q\right), \tag{3.35}$$

and the I and Q integrated spread symbols are $\boldsymbol{x}_{nI}^{(i)} = \frac{\mathbf{W}^{(i)}}{\|\boldsymbol{c}\|}\left(d_n^{(i)}\boldsymbol{c}_I - \hat{d}_n^{(i)}\boldsymbol{c}_Q\right)$ and $\boldsymbol{x}_{nQ}^{(i)} = \frac{\mathbf{W}^{(i)}}{\|\boldsymbol{c}\|}\left(\hat{d}_n^{(i)}\boldsymbol{c}_I + d_n^{(i)}\boldsymbol{c}_Q\right)$.

3. Orthogonal Covering Procedure

In practice, when simultaneous transmissions occur on the same carrier, the transmitted signal from each user channel are combined. The orthogonal covering technique is applied so that each user data can be separated and recovered at their receiver. The orthogonal covering is obtained by linearly combining the integrated spread matrix from *K* channels and is given by

$$\mathbf{X} = \mathbf{X}^{(1)} + \mathbf{X}^{(2)} + \cdots + \mathbf{X}^{(K)} = \sum_{i=1}^{K} \mathbf{X}^{(i)}, \tag{3.36}$$

where $\mathbf{X}$ is the combined $N \times m$ integrated spread matrix.

4. Complex De-spreading Procedure

The de-spreading operation is achieved by multiplying each column in the received signal $\mathbf{X}$ by the $N \times N$ diagonal PN matrix $\boldsymbol{C}$ defined in (3.10). Thus, the de-spread combined repeated Walsh matrix is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\| \boldsymbol{c} \|} \mathbf{X} = \frac{\boldsymbol{C}^*}{\| \boldsymbol{c} \|} \sum_{i=1}^{K} \mathbf{X}^{(i)} = \frac{\boldsymbol{C}^*}{\| \boldsymbol{c} \|} \sum_{i=1}^{K} \mathbf{W}^{(i)} \mathbf{S}^{(i)} = \frac{1}{\| \boldsymbol{c} \|^2} \sum_{i=1}^{K} \mathbf{W}^{(i)} \boldsymbol{C}^* \boldsymbol{c} \mathbf{D}^{(i)}. \tag{3.37}$$

5. Symbol Recovery Procedure

Before proceeding to the symbol recovery procedure to recover the original symbol vector, we define the $1 \times N$ repeated Walsh vector $\widetilde{\boldsymbol{w}}$ for the respective *ith* channel using *method one* as

$$\widetilde{\boldsymbol{w}}^{(i)} = \left[\underbrace{w_1^{(i)} \cdots w_1^{(i)}}_{b}, \quad \underbrace{w_2^{(i)} \cdots w_2^{(i)}}_{b}, \quad \cdots, \quad \underbrace{w_M^{(i)} \cdots w_M^{(i)}}_{b} \right]. \tag{3.38}$$
$$i = 1, 2, \ldots, K$$

Note that $\widetilde{\boldsymbol{w}}^{(i)}$ is the diagonal of the $N \times N$ matrix $\mathbf{W}^{(i)}$ in (3.32). For *method two*,

$$\widetilde{\boldsymbol{w}}^{(i)} = \left[\underbrace{\left[w_1^{(i)} \; w_2^{(i)} \cdots w_M^{(i)} \right]}_{1}, \quad \cdots, \quad \underbrace{\left[w_1^{(i)} \; w_2^{(i)} \cdots w_M^{(i)} \right]}_{b} \right]. \tag{3.39}$$
$$i = 1, 2, \ldots, K$$

Note that $\widetilde{\boldsymbol{w}}^{(i)}$ is the diagonal of the $N \times N$ matrix $\mathbf{W}^{(i)}$ in (3.33). Thus, the symbol recovery procedure is achieved by multiplying the respective repeated Walsh vector $\widetilde{\boldsymbol{w}}^{(i)}$ with $\mathbf{V}$; hence, the symbol vector $\mathbf{D}^{(i)}$ for the respective channels can be recovered via

$$\mathbf{Z}^{(i)} = \widetilde{\boldsymbol{w}}^{(i)} \mathbf{V} = \frac{\widetilde{\boldsymbol{w}}^{(i)}}{\| \boldsymbol{c} \|^2} \sum_{i=1}^{K} \mathbf{W}^{(i)} \boldsymbol{C}^* \boldsymbol{c} \mathbf{D}^{(i)}. \tag{3.40}$$

Using channel one as an example, to recover the symbol $\mathbf{D}^{(1)}$, we multiply the repeated Walsh vector $\widetilde{\boldsymbol{w}}^{(1)}$ from *method one* with $\mathbf{V}$ yielding

$$\mathbf{Z}^{(1)} = \widetilde{\boldsymbol{w}}^{(1)}\mathbf{V} = \frac{\widetilde{\boldsymbol{w}}^{(1)}}{\|\boldsymbol{c}\|^2}\left(\mathbf{W}^{(1)}\boldsymbol{C}^{*}\boldsymbol{c}\mathbf{D}^{(1)} + \cdots + \mathbf{W}^{(K)}\boldsymbol{C}^{*}\boldsymbol{c}\mathbf{D}^{(K)}\right)$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\left[\underbrace{w_1^{(1)}\cdots w_1^{(1)}}_{b}, \cdots, \underbrace{w_M^{(1)}\cdots w_M^{(1)}}_{b}\right]\left(\left[\left[\begin{bmatrix} w_1^{(1)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_1^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_M^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_M^{(1)} \end{bmatrix}\begin{bmatrix} c_1^{*} & 0 & 0 & 0 \\ 0 & c_1^{*} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^{*} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}\right]\mathbf{D}^{(1)} + \cdots\right.$$

$$\left. \cdots\left[\begin{bmatrix} w_1^{(K)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_1^{(K)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_M^{(K)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_M^{(K)} \end{bmatrix}\begin{bmatrix} c_1^{*} & 0 & 0 & 0 \\ 0 & c_1^{*} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^{*} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}\right]\mathbf{D}^{(K)}\right). \tag{3.41}$$

Since the Walsh codes assigned to each channel are mutually orthogonal to one another, that is $\boldsymbol{w}^{(i)}\left(\boldsymbol{w}^{(j)}\right)^{t} = 0 \;\; for \;\; j \neq i$, it can be shown that

$$\mathbf{Z}^{(1)} = \frac{1}{\|\boldsymbol{c}\|^2}\left[\underbrace{\left(w_1^{(1)}\right)^2\cdots\left(w_1^{(1)}\right)^2}_{b}, \cdots, \underbrace{\left(w_M^{(1)}\right)^2\cdots\left(w_M^{(1)}\right)^2}_{b}\right]\begin{bmatrix} |c_1|^2 \\ |c_2|^2 \\ \vdots \\ |c_N|^2 \end{bmatrix}\mathbf{D}^{(1)} + \cdots$$

$$+ \frac{1}{\|\boldsymbol{c}\|^2}\left[\underbrace{w_1^{(1)}w_1^{(K)}\cdots w_1^{(1)}w_1^{(K)}}_{b}, \cdots, \underbrace{w_M^{(1)}w_M^{(K)}\cdots w_M^{(1)}w_M^{(K)}}_{b}\right]\begin{bmatrix} |c_1|^2 \\ |c_2|^2 \\ \vdots \\ |c_N|^2 \end{bmatrix}\mathbf{D}^{(K)} \tag{3.42}$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\left\{\left(w_1^{(1)}\right)^2\left(|c_1|^2+\cdots+|c_b|^2\right)+\left(w_2^{(1)}\right)^2\left(|c_{b+1}|^2+\cdots+|c_{2b}|^2\right)+\cdots+\left(w_M^{(1)}\right)^2\left(|c_{N-b+1}|^2+\cdots+|c_N|^2\right)\right\}\mathbf{D}^{(1)}+\cdots$$

$$\frac{1}{\|\boldsymbol{c}\|^2}\underbrace{\left\{w_1^{(1)}w_1^{(K)}\left(|c_1|^2+\cdots+|c_b|^2\right)+w_2^{(1)}w_2^{(K)}\left(|c_{b+1}|^2+\cdots+|c_{2b}|^2\right)+\cdots+w_M^{(1)}w_M^{(K)}\left(|c_{N-b+1}|^2+\cdots+|c_N|^2\right)\right\}}_{0}\mathbf{D}^{(K)}$$

$$= \frac{\|\boldsymbol{c}\|^2}{\|\boldsymbol{c}\|^2}\mathbf{D}^{(1)} = \mathbf{D}^{(1)}.$$

Note that $\left(w_1^{(1)}\right)^2 = \cdots\left(w_M^{(1)}\right)^2 = 1$, $|c_1|^2 = \cdots |c_N|^2$ and $w_1^{(1)}w_1^{(K)} + \cdots + w_M^{(1)}w_M^{(K)} = 0$ via $\boldsymbol{w}^{(1)}\left(\boldsymbol{w}^{(K)}\right)^{t} = 0 \;\; for \; K \neq 1$. Therefore, only the symbols for channel one are recovered. Using repeated Walsh vector $\widetilde{\boldsymbol{w}}^{(1)}$ from *method two*, we obtained the recovered symbol vector

$$\mathbf{Z}^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{V} = \frac{\tilde{\boldsymbol{w}}^{(1)}}{\|\boldsymbol{c}\|^2}\left(\mathbf{W}^{(1)}\boldsymbol{C}^*\boldsymbol{c}\mathbf{D}^{(1)} + \cdots + \mathbf{W}^{(K)}\boldsymbol{C}^*\boldsymbol{c}\mathbf{D}^{(K)}\right)$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right], \cdots, \left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]\left(\left[\left[\begin{array}{ccccccc} w_1^{(1)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_M^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_1^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_M^{(1)} \end{array}\right]\left[\begin{array}{cccc} c_1^* & 0 & 0 & 0 \\ 0 & c_1^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{array}\right]\left[\begin{array}{c} c_1 \\ c_2 \\ \vdots \\ c_N \end{array}\right]\mathbf{D}^{(1)} + \cdots\right.\right.$$

$$\left.\left. \cdots\left[\left[\begin{array}{ccccccc} w_1^{(K)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_M^{(K)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_1^{(K)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & w_M^{(K)} \end{array}\right]\left[\begin{array}{cccc} c_1^* & 0 & 0 & 0 \\ 0 & c_1^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{array}\right]\left[\begin{array}{c} c_1 \\ c_2 \\ \vdots \\ c_N \end{array}\right]\mathbf{D}^{(K)}\right]\right). \quad (3.43)$$

Again, it can be shown that only the symbols for channel one are recovered by

$$\mathbf{Z}^{(1)} = \frac{1}{\|\boldsymbol{c}\|^2}\left[\left[\left(w_1^{(1)}\right)^2\left(w_2^{(1)}\right)^2\cdots\left(w_M^{(1)}\right)^2\right], \cdots, \left[\left(w_1^{(1)}\right)^2\left(w_2^{(1)}\right)^2\cdots\left(w_M^{(1)}\right)^2\right]\right]\left[\begin{array}{c} |c_1|^2 \\ |c_2|^2 \\ \vdots \\ |c_N|^2 \end{array}\right]\mathbf{D}^{(1)} + \cdots$$

$$+ \frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}w_1^{(K)}\, w_2^{(1)}w_2^{(K)}\cdots w_M^{(1)}w_M^{(K)}\right], \cdots, \; w_1^{(1)}w_1^{(K)}\, w_2^{(1)}w_2^{(K)}\cdots w_M^{(1)}w_M^{(K)}\right]\left[\begin{array}{c} |c_1|^2 \\ |c_2|^2 \\ \vdots \\ |c_N|^2 \end{array}\right]\mathbf{D}^{(K)}$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\left\{\left(w_1^{(1)}\right)^2|c_1|^2 + \cdots\left(w_M^{(1)}\right)^2|c_M|^2 + \cdots + \left(w_1^{(1)}\right)^2|c_{N-M+1}|^2 + \cdots\left(w_M^{(1)}\right)^2|c_N|^2\right\}\mathbf{D}^{(1)} + \cdots \quad (3.44)$$

$$\frac{1}{\|\boldsymbol{c}\|^2}\left\{\underbrace{\left(w_1^{(1)}w_1^{(K)}|c_1|^2 + \cdots w_M^{(1)}w_M^{(K)}|c_M|^2 + \cdots + w_1^{(1)}w_1^{(K)}|c_{N-M+1}|^2 + \cdots w_M^{(1)}w_M^{(K)}|c_N|^2\right)}_{0}\right\}\mathbf{D}^{(K)}$$

$$= \frac{\|\boldsymbol{c}\|^2}{\|\boldsymbol{c}\|^2}\mathbf{D}^{(1)} = \mathbf{D}^{(1)}.$$

Note that $\left(w_1^{(1)}\right)^2 = \cdots\left(w_M^{(1)}\right)^2 = 1$, $|c_1|^2 = \cdots |c_N|^2$ and $w_1^{(1)}w_1^{(K)} + \cdots + w_M^{(1)}w_M^{(K)} = 0$ via $\boldsymbol{w}^{(1)}\left(\boldsymbol{w}^{(K)}\right)^t = 0$ *for* $K \neq 1$. Therefore, only the symbols for channel one are recovered. This yields the final result in (3.44).

6. Example 2

Here, an example using two channels is shown. Assuming QPSK signals, consider a row of 2 complex symbols to be transmitted for channel one be $\mathbf{D}^{(1)} = \begin{bmatrix} 1+j & -1-j \end{bmatrix}$ and channel two be $\mathbf{D}^{(2)} = \begin{bmatrix} -1+j & 1-j \end{bmatrix}$. Let the complex PN vector $\boldsymbol{c} = \begin{bmatrix} -1-j & 1+j & 1+j & -1-j \end{bmatrix}^t$, and assume the two channels are spread with the same spreading code. Furthermore, let the Walsh codes for channels one and two be $\boldsymbol{w}^{(1)} = \begin{bmatrix} 1 & 1 \end{bmatrix}^t$ and $\boldsymbol{w}^{(2)} = \begin{bmatrix} 1 & -1 \end{bmatrix}^t$ respectively. The spreading factors in this case are $N = 4$ and $M = 2$, hence, $b = 2$. The complex spreading matrices for channels one and two are given by

$$\mathbf{S}^{(1)} = \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D}^{(1)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -1-j \\ 1+j \\ 1+j \\ -1-j \end{bmatrix}\begin{bmatrix} 1+j & -1-j \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} \tag{3.45}$$

$$\mathbf{S}^{(2)} = \frac{\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D}^{(2)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -1-j \\ 1+j \\ 1+j \\ -1-j \end{bmatrix}\begin{bmatrix} -1+j & 1-j \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

Given $b = 2$, the Walsh vectors for channels one and two have to be repeated two times to meet the chip rate of the PN code. The 4×4 diagonal Walsh matrix $\mathbf{W}$ is defined such that each Walsh chip can be multiplied with each spread chip. Thus, the repeated Walsh vector is given by one of the two methods:

<u>*Method one*</u>:

$$\mathbf{W}^{(1)} = diagonal\begin{bmatrix} 1 & \underline{1} & 1 & \underline{1} \end{bmatrix} \qquad \mathbf{W}^{(2)} = diagonal\begin{bmatrix} 1 & \underline{1} & -1 & \underline{-1} \end{bmatrix}, \tag{3.46}$$

<u>*Method two*</u>:

$$\mathbf{W}^{(1)} = diagonal\begin{bmatrix} 1 & 1 & \underline{1} & \underline{1} \end{bmatrix} \qquad \mathbf{W}^{(2)} = diagonal\begin{bmatrix} 1 & -1 & \underline{1} & \underline{-1} \end{bmatrix}. \tag{3.47}$$

With *method one*, in order to apply Walsh coding, the spread matrices $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$ are multiplied with the respective diagonal Walsh matrix, and the integrated spread matrices are obtained as

$$\mathbf{X}^{(1)} = \mathbf{W}^{(1)}\mathbf{S}^{(1)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \underline{1} \end{bmatrix}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} \quad (3.48)$$

$$\mathbf{X}^{(2)} = \mathbf{W}^{(2)}\mathbf{S}^{(2)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \underline{1} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \underline{-1} \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 2 & -2 \\ -2 & 2 \end{bmatrix}.$$

Orthogonal covering is achieved by combining the integrated spread matrix for channels one and two, which is given as

$$\mathbf{X} = \mathbf{X}^{(1)} + \mathbf{X}^{(2)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} + \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ 2 & -2 \\ -2 & 2 \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2-2j & -2+2j \\ -2+2j & 2-2j \\ 2+2j & -2-2j \\ -2-2j & 2+2j \end{bmatrix}. \quad (3.49)$$

To perform de-spreading, the $N \times N$ diagonal PN matrix $\boldsymbol{C}$ is defined in (3.10). Thus, $\boldsymbol{C} = diagonal\begin{bmatrix} -1-j & 1+j & 1+j & -1-j \end{bmatrix}$ and the complex conjugate of $\boldsymbol{C}$ are multiplied with the orthogonal covering matrix $\mathbf{X}$. Thus, the de-spread combined repeated Walsh matrix is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X} = \frac{1}{8}\begin{bmatrix} -1+j & 0 & 0 & 0 \\ 0 & 1-j & 0 & 0 \\ 0 & 0 & 1-j & 0 \\ 0 & 0 & 0 & -1+j \end{bmatrix}\begin{bmatrix} 2-2j & -2+2j \\ -2+2j & 2-2j \\ 2+2j & -2-2j \\ -2-2j & 2+2j \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4j & -4j \\ 4j & -4j \\ 4 & -4 \\ 4 & -4 \end{bmatrix}. \quad (3.50)$$

Before we proceed to recover the symbols for channels one and two, we have to define the $1 \times N$ repeated Walsh vector $\tilde{\boldsymbol{w}}^{(i)}$ for channels one and two as follows:

$$\tilde{\boldsymbol{w}}^{(1)} = \begin{bmatrix} 1 & \underline{1} & 1 & \underline{1} \end{bmatrix} \qquad \tilde{\boldsymbol{w}}^{(2)} = \begin{bmatrix} 1 & \underline{1} & -1 & -\underline{1} \end{bmatrix}. \tag{3.51}$$

Thus, to recover the symbols for channel one, the repeated Walsh vector $\tilde{\boldsymbol{w}}^{(1)}$ is multiplied with **V** yielding

$$\begin{aligned} \mathbf{Z}^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{V} &= \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4j & -4j \\ 4j & -4j \\ 4 & -4 \\ 4 & -4 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} 8+8j & -8-8j \end{bmatrix} = \begin{bmatrix} 1+j & -1-j \end{bmatrix} = \mathbf{D}^{(1)}. \end{aligned} \tag{3.52}$$

The symbols for channel two is recovered via the following:

$$\begin{aligned} \mathbf{Z}^{(2)} = \tilde{\boldsymbol{w}}^{(2)}\mathbf{V} &= \frac{1}{8}\begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 4j & -4j \\ 4j & -4j \\ 4 & -4 \\ 4 & -4 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} -8+8j & 8-8j \end{bmatrix} = \begin{bmatrix} -1+j & 1-j \end{bmatrix} = \mathbf{D}^{(2)}. \end{aligned} \tag{3.53}$$

With *method two*, the integrated spread matrices are given by

$$\begin{aligned} \mathbf{X}^{(1)} = \mathbf{W}^{(1)}\mathbf{S}^{(1)} &= \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 \\ 0 & 0 & 0 & \underline{1} \end{bmatrix}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} \\ \mathbf{X}^{(2)} = \mathbf{W}^{(2)}\mathbf{S}^{(2)} &= \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \underline{1} & 0 \\ 0 & 0 & 0 & -\underline{1} \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ 2 & -2 \\ -2 & 2 \\ -2 & 2 \end{bmatrix}. \end{aligned} \tag{3.54}$$

The orthogonal covering matrix is given as

$$\mathbf{X} = \mathbf{X}^{(1)} + \mathbf{X}^{(2)} = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} + \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 & -2 \\ 2 & -2 \\ -2 & 2 \\ -2 & 2 \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2-2j & -2+2j \\ 2+2j & -2-2j \\ -2+2j & 2-2j \\ -2-2j & 2+2j \end{bmatrix}. \quad (3.55)$$

The $N \times N$ diagonal PN matrix $\boldsymbol{C}$ is defined in (3.10). Thus, $\boldsymbol{C} = diagonal\begin{bmatrix} -1-j & 1+j & 1+j & -1-j \end{bmatrix}$. Multiplying the complex conjugate of $\boldsymbol{C}$ with **X,** we obtain the de-spread combined repeated Walsh matrix given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X} = \frac{1}{8}\begin{bmatrix} -1+j & 0 & 0 & 0 \\ 0 & 1-j & 0 & 0 \\ 0 & 0 & 1-j & 0 \\ 0 & 0 & 0 & -1+j \end{bmatrix}\begin{bmatrix} 2-2j & -2+2j \\ 2+2j & -2-2j \\ -2+2j & 2-2j \\ -2-2j & 2+2j \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4j & -4j \\ 4 & -4 \\ 4j & -4j \\ 4 & -4 \end{bmatrix}. \quad (3.56)$$

The $1 \times N$ "repeated" Walsh vector $\tilde{\boldsymbol{w}}^{(i)}$ for channels one and two are defined as

$$\tilde{\boldsymbol{w}}^{(1)} = \begin{bmatrix} 1 & 1 & \underline{1} & \underline{1} \end{bmatrix} \qquad \tilde{\boldsymbol{w}}^{(2)} = \begin{bmatrix} 1 & -1 & \underline{1} & -\underline{1} \end{bmatrix}. \quad (3.57)$$

Finally, the symbols for channel one are recovered as

$$\begin{aligned} \mathbf{Z}^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{V} &= \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4j & -4j \\ 4 & -4 \\ 4j & -4j \\ 4 & -4 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} 8+8j & -8-8j \end{bmatrix} \\ &= \begin{bmatrix} 1+j & -1-j \end{bmatrix} \\ &= \mathbf{D}^{(1)}, \end{aligned} \quad (3.58)$$

and the symbols for channel two are recovered via

$$\begin{aligned}\mathbf{Z}^{(2)} = \tilde{\boldsymbol{w}}^{(2)}\mathbf{V} &= \frac{1}{8}\begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 4j & -4j \\ 4 & -4 \\ 4j & -4j \\ 4 & -4 \end{bmatrix} \\ &= \frac{1}{8}\begin{bmatrix} -8+8j & 8-8j \end{bmatrix} \\ &= \begin{bmatrix} -1+j & 1-j \end{bmatrix} \\ &= \mathbf{D}^{(2)}.\end{aligned} \tag{3.59}$$

D. SUMMARY

In this chapter, we discussed the procedures for complex spreading with orthogonal covering for fixed data rate channels. The integrated spreading procedure was covered in Section B, where the use of orthogonal covering of spread spectrum signals was illustrated. The use of repeated Walsh codes/vector to meet the common chip rate of the PN codes for the integrated spreading process was explained. The integrated spreading with orthogonal covering procedure for multiuser application was covered in Section C. Numerical examples were also provided at the end of Sections B and C.

IV. COMPLEX SPREAD SPECTRUM SIGNALING WITH ORTHOGONAL COVERING FOR MULTI-RATE APPLICATIONS

A. INTRODUCTION

In Chapter III, the Walsh-PN complex spreading and de-spreading procedures for fixed data rate were covered. The orthogonal covering technique in Chapter III can only be applied to user channels with identical symbol rate. For multi-rate transmissions, variable length orthogonal sequences must be employed. Different spread factors are required for user channels with different data rates. Here, the variable length orthogonal covering procedure for multiuser channels with multi-rate transmissions are explained. Consider two user channels as shown in Figure 18; channel one has a symbol rate of R_s and channel two has a symbol rate of $2R_s$.

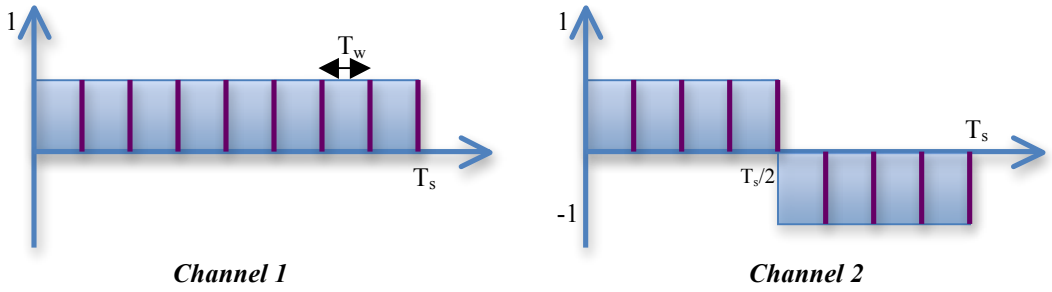

Figure 18. Multi-rate channels with repeated symbols are shown.

Note that one symbol of channel one covers the same duration as two symbols of channel two. To design a common rate for the two channels, variable length Walsh sequences are used. Channel one employs a Walsh sequence with spreading factor of eight chips, and the chip rate is $8R_s$. Channel two employs a Walsh sequence with spreading factor of four chips and the chip rate is $2R_s \times 4 = 8R_s$. Hence, the two channels meet the requirement of common chip rate and spread bandwidth. A lower bit rate channel is assigned a longer Walsh sequence, and the higher bit rate channel is assigned a shorter Walsh sequence. To generalize to the multiuser case of *K* channels with multi-rates transmissions, the orthogonal covering symbol can be written as

$$\mathbf{S} = \boldsymbol{w}_n^{\langle 1\rangle} d_l^{\langle 1\rangle} + \overbrace{\boldsymbol{w}_{n-1}^{\langle 2\rangle} d_{l+1}^{\langle 2\rangle}} + \cdots + \overbrace{\boldsymbol{w}_{n-(k-1)}^{\langle k\rangle} d_{l+(k-1)}^{\langle k\rangle}}, \tag{4.1}$$

where $\boldsymbol{w}_n^{\langle i\rangle}$ represents Walsh code *i* with spread factor 2^n, and $d_l^{\langle i\rangle}$ represents the data symbol from channel *i* with a data rate of $2^l R_s$, where *n* and *l* are integers. From the above expression, Walsh code $\boldsymbol{w}_n^{\langle 1\rangle}$ is twice the length of $\boldsymbol{w}_{n-1}^{\langle 2\rangle}$, and the symbol in $d_l^{\langle 1\rangle}$ is transmitted at half the rate of $d_{l+1}^{\langle 2\rangle}$ and so on for the *K* channels. The first term is always for the channel with the lowest bit rate, i.e., lowest *l* and highest *n*. The Walsh-data symbols on the second term onwards are concatenated (denoted by $\overbrace{|\bullet|}$) to meet the common rate of the predecessor sequence. This procedure is explained in the next section. The number of Walsh-data channels is limited to the quantity of unique Walsh codes. As mentioned before, there are two ways to perform the integrated spreading. Here, the latter method is used where Walsh coding is performed to explain the concatenation of Walsh vector for multi-rate transmission.

1. Walsh Coding Procedure

Two channels are used to illustrate the procedures. Let $\mathbf{D}^{(1)}$ be a row vector of complex m_1 symbols to be transmitted in channel one, and $\mathbf{D}^{(2)}$ be a row vector of complex m_2 symbols to be transmitted in channel two. Let the symbol rates for channel one and channel two be R_s and $2R_s$, respectively. The transmission data rate for channel two is twice that of channel one, hence, $m_2 = 2m_1$. Note that the data rate factor must be a power of two, i.e., 2, 4, 8, 16... To bring the two channels to a common spreading rate, we have to assign two variable length orthogonal Walsh sequences. Let the $M \times 1$ Walsh vector for channel one be $\boldsymbol{w}^{(1)}$ and the $L \times 1$ Walsh vector for channel two be $\boldsymbol{w}^{(2)}$, where $M = 2^l L$. Hence, in this case *M* is twice the length of *L*, i.e., *M* = 2*L*. The Walsh-data matrices $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$ for channel one and channel two are given by

$$\mathbf{S}^{(1)} = \boldsymbol{w}^{(1)}\mathbf{D}^{(1)} \qquad \mathbf{S}^{(2)} = \boldsymbol{w}^{(2)}\mathbf{D}^{(2)}, \tag{4.2}$$

where the $M \times m_1$ matrix $\mathbf{S}^{(1)}$ and the $L \times m_2$ matrix $\mathbf{S}^{(2)}$ are expressed in the form

$$\begin{aligned}
\mathbf{S}^{(1)} &= \begin{bmatrix} w_1^{(1)} \\ w_2^{(1)} \\ \vdots \\ w_M^{(1)} \end{bmatrix} \begin{bmatrix} D_1^{(1)} & D_2^{(1)} & \cdots & D_{m_1}^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_1^{(1)} & \boldsymbol{s}_2^{(1)} & \cdots & \boldsymbol{s}_{m_1}^{(1)} \end{bmatrix} \\
\mathbf{S}^{(2)} &= \begin{bmatrix} w_1^{(2)} \\ w_2^{(2)} \\ \vdots \\ w_L^{(2)} \end{bmatrix} \begin{bmatrix} D_1^{(2)} & D_2^{(2)} & \cdots & D_{m_2}^{(2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_1^{(2)} & \boldsymbol{s}_2^{(2)} & \cdots & \boldsymbol{s}_{m_2}^{(2)} \end{bmatrix},
\end{aligned} \tag{4.3}$$

and where $\boldsymbol{s}_n^{(1)}$ is a $M \times 1$ Walsh-data vector and $\boldsymbol{s}_n^{(2)}$ is a $L \times 1$ Walsh-data vector. For an arbitrary $D_n^{(i)}$, $\boldsymbol{s}_n^{(i)} = D_n^{(i)} \boldsymbol{w}^{(i)} = \left(d_n^{(i)} + j\hat{d}_n^{(i)} \right) \boldsymbol{w}^{(i)}$. Thus, the complex Walsh-data vector takes the form $\boldsymbol{s}_n^{(i)} = \boldsymbol{s}_{nI}^{(i)} + j\boldsymbol{s}_{nQ}^{(i)}$ where the I and Q Walsh-data vectors for channels one and two are given by

$$\underbrace{\boldsymbol{s}_{nI}^{(1)} = d_n^{(1)} \boldsymbol{w}^{(1)} \qquad \boldsymbol{s}_{nQ}^{(1)} = \hat{d}_n^{(1)} \boldsymbol{w}^{(1)}}_{Channel\,1}, \qquad \underbrace{\boldsymbol{s}_{nI}^{(2)} = d_n^{(2)} \boldsymbol{w}^{(2)} \qquad \boldsymbol{s}_{nQ}^{(2)} = \hat{d}_n^{(2)} \boldsymbol{w}^{(2)}}_{Channel\,2}. \tag{4.4}$$

2. Repeating the Walsh-Data Vectors

Let the PN code $\boldsymbol{c}$ be a $N \times 1$ column vector. Given that $N = bM$, the Walsh-data vectors in $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$ have to be repeated b times. Hence, the repeated Walsh matrices are given by

$$\tilde{\mathbf{S}}^{(1)} = \begin{bmatrix} \underbrace{\mathbf{S}^{(1)}}_{1} \\ \underbrace{\mathbf{S}^{(1)}}_{2} \\ \vdots \\ \underbrace{\mathbf{S}^{(1)}}_{b} \end{bmatrix} = \underbrace{\begin{bmatrix} w_1^{(1)}D_1^{(1)} & w_1^{(1)}D_2^{(1)} & \cdots & w_1^{(1)}D_{m_1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \underbrace{w_M^{(1)}D_1^{(1)}}_{1} & \underbrace{w_M^{(1)}D_2^{(1)}}_{1} & \cdots & \underbrace{w_M^{(1)}D_{m_1}^{(1)}}_{1} \\ \vdots & \vdots & \vdots & \vdots \\ w_1^{(1)}D_1^{(1)} & w_1^{(1)}D_2^{(1)} & \cdots & w_1^{(1)}D_{m_1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \underbrace{w_M^{(1)}D_1^{(1)}}_{b} & \underbrace{w_1^{(1)}D_2^{(1)}}_{b} & \cdots & \underbrace{w_M^{(1)}D_{m_1}^{(1)}}_{b} \end{bmatrix}}_{\left(\tilde{\mathbf{w}}^{(1)}\right)^t \mathbf{D}^{(1)}}$$

$$\tilde{\mathbf{S}}^{(2)} = \begin{bmatrix} \underbrace{\mathbf{S}^{(2)}}_{1} \\ \underbrace{\mathbf{S}^{(2)}}_{2} \\ \vdots \\ \underbrace{\mathbf{S}^{(2)}}_{b} \end{bmatrix} = \underbrace{\begin{bmatrix} w_1^{(2)}D_1^{(2)} & w_1^{(2)}D_2^{(2)} & \cdots & w_1^{(2)}D_{m_2}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \underbrace{w_L^{(2)}D_1^{(2)}}_{1} & \underbrace{w_L^{(2)}D_2^{(2)}}_{1} & \cdots & \underbrace{w_L^{(2)}D_{m_2}^{(2)}}_{1} \\ \vdots & \vdots & \vdots & \vdots \\ w_1^{(2)}D_1^{(2)} & w_1^{(2)}D_2^{(2)} & \cdots & w_1^{(2)}D_{m_2}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \underbrace{w_L^{(2)}D_1^{(2)}}_{b} & \underbrace{w_L^{(2)}D_2^{(2)}}_{b} & \cdots & \underbrace{w_L^{(2)}D_{m_2}^{(2)}}_{b} \end{bmatrix}}_{\left(\tilde{\mathbf{w}}^{(2)}\right)^t \mathbf{D}^{(2)}}, \tag{4.5}$$

where $\tilde{\mathbf{S}}^{(1)}$ is now an $N \times m_1$ matrix and $\tilde{\mathbf{S}}^{(2)}$ is an $\frac{N}{2^l} \times m_2$ matrix. Thus, in order to perform orthogonal covering, $\tilde{\mathbf{S}}^{(2)}$ has to have the same common chip rate as $\tilde{\mathbf{S}}^{(1)}$, i.e., same dimension.

3. Concatenating the Walsh-Data Vectors

Since the transmission rate for channel two is twice that of channel one i.e., $l = 1$, we concatenate the repeated Walsh-data vectors such that the columns of $\tilde{\mathbf{S}}^{(2)}$ have the same length, i.e., same chip rate as the columns of $\tilde{\mathbf{S}}^{(1)}$. The explanation of the concatenate procedure is depicted below for $2^l = 2$:

$$\tilde{\mathbf{S}}^{(2)} = \begin{bmatrix} w_1^{(2)} D_1^{(2)} & w_1^{(2)} D_2^{(2)} & \cdots & w_1^{(2)} D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ w_L^{(2)} D_1^{(2)} & w_L^{(2)} D_2^{(2)} & \cdots & w_L^{(2)} D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ w_1^{(2)} D_{\frac{m_2}{2}+1}^{(2)} & w_1^{(2)} D_{\frac{m_2}{2}+2}^{(2)} & \cdots & w_1^{(2)} D_{m_2}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ w_L^{(2)} D_{\frac{m_2}{2}+1}^{(2)} & w_L^{(2)} D_{\frac{m_2}{2}+2}^{(2)} & \cdots & w_L^{(2)} D_{m_2}^{(2)} \end{bmatrix}. \tag{4.6}$$

Thus, $\tilde{\mathbf{S}}^{(2)}$ is now an $N \times m_1$ matrix, and each column consists of two concatenated repeated Walsh-data vectors.

4. Complex Spreading Procedure

To perform spreading, each column vector of the repeated Walsh-data matrix has to be multiplied by the PN code ***c***. We have to define the $N \times N$ diagonal PN matrix given by

$$\boldsymbol{C} = diagonal\left[\; c_1, \quad c_2, \quad \cdots, \quad c_N \; \right], \tag{4.7}$$

and the integrated spread matrices for channels one and two are given by

$$\mathbf{X}^{(1)} = \frac{\boldsymbol{C}}{\| \boldsymbol{c} \|} \tilde{\mathbf{S}}^{(1)} \qquad \mathbf{X}^{(2)} = \frac{\boldsymbol{C}}{\| \boldsymbol{c} \|} \tilde{\mathbf{S}}^{(2)}, \tag{4.8}$$

where $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are an $N \times m_1$ integrated spread matrices.

5. Orthogonal Covering Procedure

The orthogonal covering is obtained by linearly combining the integrated spread matrices from channel one and channel two and is given by

$$\mathbf{X} = \mathbf{X}^{(1)} + \mathbf{X}^{(2)} = \sum_{i=1}^{2} \mathbf{X}^{(i)}, \tag{4.9}$$

where $\mathbf{X}$ is the combined $N \times m_1$ integrated spread matrix.

6. Complex De-spreading Procedure

The de-spreading is obtained by multiplying $\mathbf{X}$ with the complex conjugate of the $N \times N$ PN matrix $\boldsymbol{C}$. Thus,

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\sum_{i=1}^{2}\mathbf{X}^{(i)} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\left(\frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\tilde{\mathbf{S}}^{(1)} + \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\tilde{\mathbf{S}}^{(2)}\right) = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\left(\tilde{\mathbf{S}}^{(1)} + \tilde{\mathbf{S}}^{(2)}\right), \qquad (4.10)$$

where $\mathbf{V}$ is a $N \times m_1$ matrix which reduces to

$$\mathbf{V} = \frac{1}{\|\boldsymbol{c}\|^2}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}}_{N\times N}\left(\underbrace{\begin{bmatrix} w_1^{(1)}D_1^{(1)} & \cdots & w_1^{(1)}D_{m_1}^{(1)} \\ \vdots & \ddots & \vdots \\ w_M^{(1)}D_1^{(1)} & \cdots & w_M^{(1)}D_{m_1}^{(1)} \\ \vdots & \vdots & \vdots \\ w_1^{(1)}D_1^{(1)} & \cdots & w_1^{(1)}D_{m_1}^{(1)} \\ \vdots & \ddots & \vdots \\ w_M^{(1)}D_1^{(1)} & \cdots & w_M^{(1)}D_{m_1}^{(1)} \end{bmatrix}}_{\tilde{\mathbf{S}}^{(1)},\ N\times m_1} + \underbrace{\begin{bmatrix} w_1^{(2)}D_1^{(2)} & \cdots & w_1^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \ddots & \vdots \\ w_L^{(2)}D_1^{(2)} & \cdots & w_L^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \vdots & \vdots \\ w_1^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_1^{(2)}D_{m_2}^{(2)} \\ \vdots & \ddots & \vdots \\ w_L^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_L^{(2)}D_{m_2}^{(2)} \end{bmatrix}}_{\tilde{\mathbf{S}}^{(2)},\ N\times m_1}\right) \qquad (4.11)$$

$$= \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1m_1} \\ v_{21} & v_{22} & \cdots & v_{2m_1} \\ \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & \cdots & v_{Nm_1} \end{bmatrix}.$$

7. Symbol Recovery Procedure

To recover the original symbols for channel one, we can define the $1 \times N$ "repeated" Walsh vector $\tilde{\boldsymbol{w}}^{(1)}$ given as

$$\tilde{\boldsymbol{w}}^{(1)} = \left[\ \underbrace{\left[w_1^{(1)} \cdots w_M^{(1)}\right]}_{1},\ \cdots,\ \underbrace{\left[w_1^{(1)} \cdots w_M^{(1)}\right]}_{b}\ \right], \qquad (4.12)$$

and the symbol recovery procedure for channel one is achieved by multiplying the repeated Walsh vector $\widetilde{\boldsymbol{w}}^{(1)}$ with $\mathbf{V}$. Hence, the symbol vector $\mathbf{D}^{(1)}$ is recovered as follows:

$$\mathbf{Z}^{(1)} = \widetilde{\boldsymbol{w}}^{(1)}\mathbf{V} = \frac{\widetilde{\boldsymbol{w}}^{(1)}}{\|\boldsymbol{c}\|^2}\boldsymbol{C}^*\boldsymbol{C}\left(\tilde{\mathbf{S}}^{(1)} + \tilde{\mathbf{S}}^{(2)}\right)$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\underbrace{\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right], \cdots, \left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]}_{1\times N}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}}_{N\times N}\overbrace{\begin{bmatrix} w_1^{(1)} \\ \vdots \\ w_M^{(1)} \\ \vdots \\ w_1^{(1)} \\ \vdots \\ w_M^{(1)} \end{bmatrix}}^{\tilde{\mathbf{s}}^{(1)}}_{N\times 1}\mathbf{D}^{(1)} +$$

$$\frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right], \cdots, \left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}\overbrace{\begin{bmatrix} w_1^{(2)}D_1^{(2)} & \cdots & w_1^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \ddots & \vdots \\ w_L^{(2)}D_1^{(2)} & \cdots & w_L^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ w_1^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & \vdots \\ \vdots & \ddots & w_1^{(2)}D_{m_2}^{(2)} \\ w_L^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_L^{(2)}D_{m_2}^{(2)} \end{bmatrix}}^{\tilde{\mathbf{s}}^{(2)}} \tag{4.13}$$

$$= \frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right], \cdots, \left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]\begin{bmatrix} |c_1|^2 & 0 & 0 & 0 \\ 0 & |c_2|^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & |c_N|^2 \end{bmatrix}\begin{bmatrix} w_1^{(1)} \\ \vdots \\ w_M^{(1)} \\ \vdots \\ w_1^{(1)} \\ \vdots \\ w_M^{(1)} \end{bmatrix}\mathbf{D}^{(1)} +$$

$$\frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right], \cdots, \left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]\begin{bmatrix} |c_1|^2 & 0 & 0 & 0 \\ 0 & |c_2|^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & |c_N|^2 \end{bmatrix}\begin{bmatrix} w_1^{(2)}D_1^{(2)} & \cdots & w_1^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ \vdots & \ddots & \vdots \\ w_L^{(2)}D_1^{(2)} & \cdots & w_L^{(2)}D_{\frac{m_2}{2}}^{(2)} \\ w_1^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & \vdots \\ \vdots & \ddots & w_1^{(2)}D_{m_2}^{(2)} \\ w_L^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_L^{(2)}D_{m_2}^{(2)} \end{bmatrix}.$$

Since $\tilde{\mathbf{S}}^{(2)}$ is Walsh coded with $\boldsymbol{w}^{(2)}$, it is known that (as explained in Chapter III) the product of two different Walsh sequence is zero. Hence, it can be shown that

$$\mathbf{Z}^{(1)} = \frac{1}{\|\boldsymbol{c}\|^2}\left[\left(w_1^{(1)}\right)^2 |c_1|^2 + \cdots \left(w_M^{(1)}\right)^2 |c_M|^2 + \cdots + \left(w_1^{(1)}\right)^2 |c_{N-M+1}|^2 + \cdots \left(w_M^{(1)}\right)^2 |c_N|^2\right]\mathbf{D}^{(1)} + \frac{1}{\|\boldsymbol{c}\|^2}\left[\underbrace{A_1}_{0} \quad \underbrace{A_2}_{0} \quad \cdots \quad \underbrace{A_{m_1}}_{0}\right], \tag{4.14}$$

and $A_i = 0$ *for* $i = 1,2,\cdots,m_1$. For example,

$$\begin{aligned} A_1 &= \frac{1}{\|\boldsymbol{c}\|^2}\left[\left[w_1^{(1)}\cdots w_M^{(1)}\right],\cdots,\left[w_1^{(1)}\cdots w_M^{(1)}\right]\right]\begin{bmatrix} |c_1|^2 & 0 & 0 & 0 \\ 0 & |c_2|^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & |c_N|^2 \end{bmatrix}\begin{bmatrix} w_1^{(2)}D_1^{(2)} \\ \vdots \\ w_L^{(2)}D_1^{(2)} \\ \vdots \\ w_1^{(2)}D_{\frac{m_2}{2}+1}^{(2)} \\ \vdots \\ w_L^{(2)}D_{\frac{m_2}{2}+1}^{(2)} \end{bmatrix} \\ &= \frac{1}{\|\boldsymbol{c}\|^2}\left\{\begin{array}{l} \underbrace{\left(w_1^{(1)}w_1^{(2)}|c_1|^2 + w_2^{(1)}w_2^{(2)}|c_2|^2 + \cdots + w_M^{(1)}w_L^{(2)}|c_M|^2\right)}_{0} D_1^{(2)} + \cdots \\ + \underbrace{\left(w_1^{(1)}w_1^{(2)}|c_{N-M+1}|^2 + w_2^{(1)}w_2^{(2)}|c_{N-M+2}|^2 + \cdots + w_M^{(1)}w_L^{(2)}|c_N|^2\right)}_{0} D_{\frac{m_2}{2}+1}^{(2)} \end{array}\right\} \\ &= 0. \end{aligned} \tag{4.15}$$

Since $\boldsymbol{w}^{(1)}$ and $\boldsymbol{w}^{(2)}$ are variable length orthogonal codes and mutually orthogonal to each other, $\tilde{\boldsymbol{w}}^{(1)}\left[\tilde{\boldsymbol{w}}^{(2)}, \tilde{\boldsymbol{w}}^{(2)}\right]^t = 0$, where $\tilde{\boldsymbol{w}}^{(1)}$ is given in (4.12) and $\tilde{\boldsymbol{w}}^{(2)}$ is given in (4.19). Hence,

$$\mathbf{Z}^{(1)} = \frac{1}{\|\boldsymbol{c}\|^2}\left(|c_1|^2 + |c_2|^2 + \cdots + |c_N|^2\right)\mathbf{D}^{(1)} = \frac{\|\boldsymbol{c}\|^2}{\|\boldsymbol{c}\|^2}\mathbf{D}^{(1)} = \mathbf{D}^{(1)}. \tag{4.16}$$

To recover the symbols for channel two, there is an additional procedure that needs to be carried out. We know that $\tilde{\mathbf{S}}^{(2)}$ is available in $\mathbf{V}$ and the received matrix $\mathbf{V}$ is a $N \times m_1$ matrix. We have to define the matrix $\mathbf{V}$ such that it becomes a $(N/2) \times m_2$ matrix. This can be done via mapping $\mathbf{V} \to \widehat{\mathbf{V}}$ and is given by

$$\widehat{\mathbf{V}} = \underbrace{\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1m_1} & v_{\left(\frac{N}{2}+1\right)1} & \cdots & v_{\left(\frac{N}{2}+1\right)m_1} \\ v_{21} & v_{22} & \cdots & v_{2m_1} & v_{\left(\frac{N}{2}+2\right)1} & \cdots & v_{\left(\frac{N}{2}+2\right)m_1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ v_{\frac{N}{2}1} & v_{\frac{N}{2}2} & \cdots & v_{\frac{N}{2}m_1} & v_{N1} & \cdots & v_{Nm_1} \end{bmatrix}}_{\frac{N}{2} \times m_2} = \widehat{\mathbf{S}}^{(1)} + \widehat{\mathbf{S}}^{(2)}. \tag{4.17}$$

Note that $m_2 = 2m_1$, therefore, the matrix $\widehat{\mathbf{V}}$ has the same number of elements as $\mathbf{V}$. For example, the $\left(\frac{N}{2}+1\right)$ to N rows of $\mathbf{V}$ is shifted and concatenated with the 1 to $N/2$ rows of $\mathbf{V}$. The matrices $\widehat{\mathbf{S}}^{(1)}$ and $\widehat{\mathbf{S}}^{(2)}$ are the resulting de-spread outputs from mapping, which are given by

$$\widehat{\mathbf{S}}^{(1)} = \frac{1}{\|\boldsymbol{c}\|^2} \underbrace{\begin{bmatrix} w_1^{(1)}\widehat{D}_1^{(1)} & w_1^{(1)}\widehat{D}_2^{(1)} & \cdots & w_1^{(1)}\widehat{D}_{m_1}^{(1)} & w_1^{(1)}\widehat{D}_1^{(1)} & \cdots & w_1^{(1)}\widehat{D}_{m_1}^{(1)} \\ w_2^{(1)}\widehat{D}_1^{(1)} & w_2^{(1)}\widehat{D}_2^{(1)} & \cdots & w_2^{(1)}\widehat{D}_{m_1}^{(1)} & w_2^{(1)}\widehat{D}_1^{(1)} & \ldots & w_2^{(1)}\widehat{D}_{m_1}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ w_M^{(1)}\widehat{D}_1^{(1)} & w_M^{(1)}\widehat{D}_2^{(1)} & \cdots & w_M^{(1)}\widehat{D}_{m_1}^{(1)} & w_M^{(1)}\widehat{D}_1^{(1)} & \cdots & w_M^{(1)}\widehat{D}_{m_1}^{(1)} \end{bmatrix}}_{\frac{N}{2} \times m_2}$$

$$\widehat{\mathbf{S}}^{(2)} = \frac{1}{\|\boldsymbol{c}\|^2} \underbrace{\begin{bmatrix} w_1^{(2)}\widehat{D}_1^{(2)} & w_1^{(1)}\widehat{D}_2^{(2)} & \cdots & w_1^{(2)}D_{\frac{m_2}{2}}^{(2)} & w_1^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_1^{(1)}\widehat{D}_{m_2}^{(2)} \\ w_2^{(2)}\widehat{D}_1^{(2)} & w_2^{(1)}\widehat{D}_2^{(2)} & \cdots & w_2^{(2)}D_{\frac{m_2}{2}}^{(2)} & w_2^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_2^{(1)}\widehat{D}_{m_2}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ w_L^{(2)}\widehat{D}_1^{(2)} & w_L^{(1)}\widehat{D}_2^{(2)} & \cdots & w_L^{(2)}D_{\frac{m_2}{2}}^{(2)} & w_L^{(2)}D_{\frac{m_2}{2}+1}^{(2)} & \cdots & w_L^{(1)}\widehat{D}_{m_2}^{(2)} \end{bmatrix}}_{\frac{N}{2} \times m_2}, \tag{4.18}$$

where $\widehat{D}^{(i)}, i=1,2$ is the version of the respective transmitted symbol scaled by the de-spreading factor $|c|^2$. Now, we define the "repeated" $1\times\frac{N}{2}$ Walsh vector given by

$$\widetilde{\boldsymbol{w}}^{(2)}=\left[\ \underbrace{\left[w_1^{(2)}\cdots w_L^{(2)}\right]}_{1},\ \cdots,\ \underbrace{\left[w_1^{(1)}\cdots w_L^{(1)}\right]}_{b}\ \right]. \tag{4.19}$$

Finally, the symbols for channel two can be recovered from

$$\begin{aligned}\mathbf{Z}^{(2)}&=2^l\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{V}}=2^l\widetilde{\boldsymbol{w}}^{(2)}\left(\widehat{\mathbf{S}}^{(1)}+\widehat{\mathbf{S}}^{(2)}\right)\\&=\underbrace{\frac{2}{\|\boldsymbol{c}\|^2}\left(\underbrace{\left[w_1^{(2)}\cdots w_L^{(2)}\right]}_{1},\ \cdots,\ \underbrace{\left[w_1^{(2)}\cdots w_L^{(2)}\right]}_{b}\right)\begin{bmatrix}w_1^{(1)}\widehat{D}_1^{(1)} & w_1^{(1)}\widehat{D}_2^{(1)} & \cdots & w_1^{(1)}\widehat{D}_{m_1}^{(1)}\\ w_2^{(1)}\widehat{D}_1^{(1)} & w_2^{(1)}\widehat{D}_2^{(1)} & \cdots & w_2^{(1)}\widehat{D}_{m_1}^{(1)}\\ \vdots & \vdots & \ddots & \vdots\\ w_M^{(1)}\widehat{D}_1^{(1)} & w_M^{(1)}\widehat{D}_2^{(1)} & \cdots & w_M^{(1)}\widehat{D}_{m_1}^{(1)}\end{bmatrix}}_{0}+\\&\underbrace{\frac{2}{\|\boldsymbol{c}\|^2}\left(\underbrace{\left[w_1^{(2)}\cdots w_L^{(2)}\right]}_{1},\ \cdots,\ \underbrace{\left[w_1^{(2)}\cdots w_L^{(2)}\right]}_{b}\right)}_{1\times\frac{N}{2}}\underbrace{\begin{bmatrix}w_1^{(2)}\widehat{D}_1^{(2)} & w_1^{(1)}\widehat{D}_2^{(2)} & \cdots & w_1^{(1)}\widehat{D}_{m_2}^{(2)}\\ w_2^{(2)}\widehat{D}_1^{(2)} & w_2^{(1)}\widehat{D}_2^{(2)} & \cdots & w_2^{(1)}\widehat{D}_{m_2}^{(2)}\\ \vdots & \vdots & \ddots & \vdots\\ w_L^{(2)}\widehat{D}_1^{(2)} & w_L^{(1)}\widehat{D}_2^{(2)} & \cdots & w_L^{(1)}\widehat{D}_{m_2}^{(2)}\end{bmatrix}}_{\frac{N}{2}\times m_2}.\end{aligned} \tag{4.20}$$

Since $\left(w_1^{(1)}\right)^2=\left(w_2^{(1)}\right)^2=\cdots\left(w_L^{(1)}\right)^2=1$ and $\widetilde{\boldsymbol{w}}^{(2)}\left(\boldsymbol{w}^{(1)}\right)^t=0$, then

$$\mathbf{Z}^{(1)}=\frac{2}{\|\boldsymbol{c}\|^2}\left(|c_1|^2+|c_2|^2+\cdots+|c_{\frac{N}{2}}|^2\right)\mathbf{D}^{(2)}=\frac{2}{\|\boldsymbol{c}\|^2}\frac{\|\boldsymbol{c}\|^2}{2}\mathbf{D}^{(2)}=\mathbf{D}^{(2)}. \tag{4.21}$$

Note that the column dimension of $\widehat{\mathbf{V}}$ is $N/2^l=N/2$. Thus, we have to multiply the scaling factor of $2^l=2$ to normalize the energy for each recovered symbols. Hence, we can see that the spreading gain for each symbol is reduced by the factor of two.

8. Example 3

Here we show an example of using two multi-rate channels. Let $\mathbf{D}^{(1)} = \begin{bmatrix} 1+j & -1-j \end{bmatrix}$ for channel one, and $\mathbf{D}^{(2)} = \begin{bmatrix} 1+j & 1+j & -1+j & 1-j \end{bmatrix}$ for channel two. The transmission rate for channel two is twice the rate for channel one. Let the complex PN vector $\boldsymbol{c} = \begin{bmatrix} 1+j & -1-j & -1-j & -1-j & 1+j & -1-j & 1+j & 1+j \end{bmatrix}$ and assume the two channels are spread with the same spreading code. Let the $M \times 1$ Walsh vector for channel one be $\boldsymbol{w}^{(1)} = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^t$ and the $L \times 1$ Walsh vector for channel two be $\boldsymbol{w}^{(2)} = \begin{bmatrix} 1 & 1 \end{bmatrix}^t$. The spreading factors in this case are $N = 8$, $M = 4$ and $L = 2$. The Walsh matrices for channels one and two are given by

$$\begin{aligned}
\mathbf{S}^{(1)} = \boldsymbol{w}^{(1)}\mathbf{D}^{(1)} &= \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1+j & -1-j \end{bmatrix} = \begin{bmatrix} 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \end{bmatrix} \\
\mathbf{S}^{(2)} = \boldsymbol{w}^{(2)}\mathbf{D}^{(2)} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1+j & 1+j & -1+j & 1-j \end{bmatrix} \\
&= \begin{bmatrix} 1+j & 1+j & -1+j & 1-j \\ 1+j & 1+j & -1+j & 1-j \end{bmatrix}.
\end{aligned} \tag{4.22}$$

Since $N = 2M$, the Walsh-data matrices are repeated two times, and we have

$$\tilde{\mathbf{S}}^{(1)} = \begin{bmatrix} \overset{\boldsymbol{w}^{(1)}D_1^{(1)}}{1+j} & \overset{\boldsymbol{w}^{(1)}D_2^{(1)}}{-1-j} \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \end{bmatrix} \qquad \tilde{\mathbf{S}}^{(2)} = \begin{bmatrix} \overset{\boldsymbol{w}^{(1)}D_1^{(2)}}{1+j} & \overset{\boldsymbol{w}^{(1)}D_2^{(2)}}{1+j} & \overset{\boldsymbol{w}^{(1)}D_3^{(2)}}{-1+j} & \overset{\boldsymbol{w}^{(1)}D_4^{(2)}}{1-j} \\ 1+j & 1+j & -1+j & 1-j \\ 1+j & 1+j & -1+j & 1-j \\ 1+j & 1+j & -1+j & 1-j \\ 1+j & 1+j & -1+j & 1-j \end{bmatrix}. \tag{4.23}$$

The transmission rate of channel two is twice that of channel one. Therefore, we concatenate the repeated Walsh vectors from $\tilde{\mathbf{S}}^{(2)}$ so that it has the same chip rate as channel one. The concatenated Walsh matrix is of the form

$$\tilde{\mathbf{S}}^{(2)} = \begin{bmatrix} \boldsymbol{w}^{(1)} D_1^{(2)} & \boldsymbol{w}^{(1)} D_2^{(2)} \\ \boldsymbol{w}^{(1)} D_3^{(2)} & \boldsymbol{w}^{(1)} D_4^{(2)} \end{bmatrix} = \begin{bmatrix} 1+j & 1+j \\ 1+j & 1+j \\ 1+j & 1+j \\ 1+j & 1+j \\ -1+j & 1-j \\ -1+j & 1-j \\ -1+j & 1-j \\ -1+j & 1-j \end{bmatrix}. \tag{4.24}$$

To perform complex spreading, each column of $\tilde{\mathbf{S}}^{(1)}$ and $\tilde{\mathbf{S}}^{(2)}$ is multiplied by the complex PN vector $\boldsymbol{c}$. Thus, we define the $N \times N$ complex PN matrix $\boldsymbol{C} = diagonal\begin{bmatrix} 1+j & -1-j & -1-j & -1-j & 1+j & -1-j & 1+j & 1+j \end{bmatrix}$. The integrated spread matrices are given by

$$\mathbf{X}^{(1)} = \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\tilde{\mathbf{S}}^{(1)} = \frac{1}{4}\begin{bmatrix} 1+j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1-j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1-j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1-j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1-j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+j \end{bmatrix}\begin{bmatrix} 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \\ 1+j & -1-j \\ -1-j & 1+j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2j & -2j \\ 2j & -2j \\ -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} \tag{4.25}$$

$$\mathbf{X}^{(2)} = \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\tilde{\mathbf{S}}^{(2)} = \frac{1}{4}\begin{bmatrix} 1+j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1-j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1-j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1-j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1-j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1+j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+j \end{bmatrix}\begin{bmatrix} 1+j & 1+j \\ 1+j & 1+j \\ 1+j & 1+j \\ 1+j & 1+j \\ -1+j & 1-j \\ -1+j & 1-j \\ -1+j & 1-j \\ -1+j & 1-j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2j & 2j \\ -2j & -2j \\ -2j & -2j \\ -2j & -2j \\ -2 & 2 \\ 2 & -2 \\ -2 & 2 \\ -2 & 2 \end{bmatrix}.$$

The orthogonal covering is achieved by linearly combining the two integrated spread matrices to get

$$\mathbf{X} = \mathbf{X}^{(1)} + \mathbf{X}^{(2)} = \frac{1}{4}\begin{bmatrix} 2j & -2j \\ 2j & -2j \\ -2j & 2j \\ 2j & -2j \\ 2j & -2j \\ 2j & -2j \\ 2j & -2j \\ -2j & 2j \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 2j & 2j \\ -2j & -2j \\ -2j & -2j \\ -2j & -2j \\ -2 & 2 \\ 2 & -2 \\ -2 & 2 \\ -2 & 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4j & 0 \\ 0 & -4j \\ -4j & 0 \\ 0 & -4j \\ -2+2j & 2-2j \\ 2+2j & -2-2j \\ -2+2j & 2-2j \\ -2-2j & 2+2j \end{bmatrix}. \quad (4.26)$$

To de-spread, we multiply the complex conjugate of the $N \times N$ PN matrix $\boldsymbol{C}$ with $\mathbf{X}$. The de-spread repeated Walsh matrix is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X} = \frac{1}{16}\begin{bmatrix} 1-j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1+j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1+j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-j \end{bmatrix}\begin{bmatrix} 4j & 0 \\ 0 & -4j \\ -4j & 0 \\ 0 & -4j \\ -2+2j & 2-2j \\ 2+2j & -2-2j \\ -2+2j & 2-2j \\ -2-2j & 2+2j \end{bmatrix} \quad (4.27)$$

$$= \frac{1}{16}\begin{bmatrix} 4+4j & 0 \\ 0 & 4+4j \\ 4+4j & 0 \\ 0 & 4+4j \\ 4j & -4j \\ -4 & 4 \\ 4j & -4j \\ -4 & 4 \end{bmatrix}.$$

To recover the original symbols in **D** for channel one, we define the repeated Walsh vector

$$\tilde{\boldsymbol{w}}^{(1)} = \begin{bmatrix} 1 & -1 & 1 & -1 & \underline{1} & \underline{-1} & \underline{1} & \underline{-1} \end{bmatrix}, \quad (4.28)$$

where the recovered symbols for channel one can be obtained via

$$\mathbf{Z}^{(1)} = \widetilde{\boldsymbol{w}}^{(1)}\mathbf{V} = \frac{1}{16}\begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 4+4j & 0 \\ 0 & 4+4j \\ 4+4j & 0 \\ 0 & 4+4j \\ 4j & -4j \\ -4 & 4 \\ 4j & -4j \\ -4 & 4 \end{bmatrix} \tag{4.29}$$

$$= \frac{1}{16}\begin{bmatrix} 16+16j & -16-16j \end{bmatrix} = \begin{bmatrix} 1+j & -1-j \end{bmatrix}.$$

To recover symbols for channel two, we have to map the de-spread 8×2 Walsh matrix $\mathbf{V}$ to a 4×4 matrix $\widehat{\mathbf{V}}$ via mapping $\mathbf{V}\rightarrow\widehat{\mathbf{V}}$ in (4.17). Thus, the new de-spread matrix is given by

$$\widehat{\mathbf{V}} = \frac{1}{16}\begin{bmatrix} 4+4j & 0 & 4j & -4j \\ 0 & 4+4j & -4 & 4 \\ 4+4j & 0 & 4j & -4j \\ 0 & 4+4j & -4 & 4 \end{bmatrix}, \tag{4.30}$$

where the repeated Walsh vector for channel two is given by

$$\widetilde{\boldsymbol{w}}^{(2)} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}. \tag{4.31}$$

Hence, the recovered symbols for channel two is given by

$$\mathbf{Z}^{(2)} = 2^{l}\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{V}} = \frac{2}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4+4j & 0 & 4j & -4j \\ 0 & 4+4j & -4 & 4 \\ 4+4j & 0 & 4j & -4j \\ 0 & 4+4j & -4 & 4 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 8+8j & 8+8j & -8+8j & 8-8j \end{bmatrix} \tag{4.32}$$

$$= \begin{bmatrix} 1+j & 1+j & -1+j & 1-j \end{bmatrix}.$$

Note the scaling factor $2^{1} = 2$ is used to obtain the unit energy for each symbols.

B. SUMMARY

In this chapter, we discussed the procedure for complex spreading with orthogonal covering for multi-rate data transmission. The Walsh sequence in the higher bit rate channel must be concatenated to meet the chip rate of the lower bit rate channel before orthogonal covering can be performed. Since the PN codes are spread across the concatenated symbols, the spreading gain for each symbol is reduced. The procedure was illustrated via numerical example.

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V. FADING, NOISE AND INTERFERENCE

A. INTRODUCTION

We covered the integrated Walsh coding and complex spreading procedures for multiuser channels in Chapter III and multi-rate applications in Chapter IV. In this chapter, we apply the orthogonal space-time block codes signals in Rayleigh fading, and noise and interference as well as the maximal ratio combining operation to our signal model. This procedure is illustrated in Figure 19.

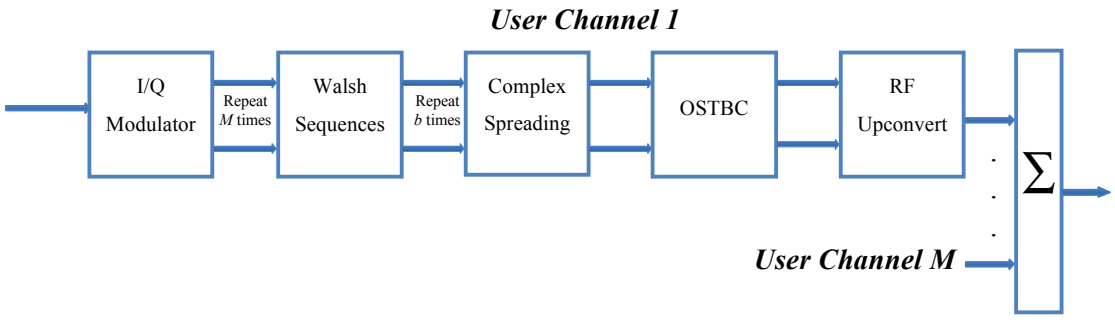

Figure 19. Integrated spreading MIMO signal with orthogonal covering procedure and OSTBC is illustrated.

In our extended channel model, the received signal may be corrupted with fading, noise and various types of interference. At the receiver, the received signal goes through de-spreading, Walsh recovery and MRC operations. In order to find the noise power of the sufficient vector statistic out of the MRC, we must analyze the noise and jammer transformation after the de-spreading, Walsh recovery and MRC operations. In Chapter II, we discussed the application of OSTBC and the MRC techniques, and now we include them in the procedures.

In a fading channel, multiple attenuated copies of the received signals are available at the input of the MRC combiner. Single-antenna receiver do not have the ability to separate the copies of the symbol. The use of transmit antenna diversity and time diversity can provide the receiver sufficient statistics to separate the copies for combining. This can be achieved by applying OSTBC in the transmitter.

B. INTEGRATED SPREADING WITH OSTBC SIGNALING IN FADING, NOISE AND JAMMING FOR A FIXED DATA RATE CHANNEL

In Chapter III, we explained the integrated spreading procedures without noise and interference. Now, we add the OSTBC to the integrated spreading matrix **X** in Chapter III. Our work is an extension of the result in [2] where orthogonal covering was not considered.

1. Received Signal with Noise and Interference

We extend the procedure in (3.7), where the $N \times m$ integrated spread matrix **X** is given as $\mathbf{X} = \mathbf{WS}$. When the signal goes through an AWGN channel and jamming, the received signal is given by

$$\mathbf{Y} = \mathbf{X}\mathbf{H}^t + \mathbf{N} + \mathbf{J} = \mathbf{W}\mathbf{S}\mathbf{H}^t + \mathbf{N} + \mathbf{J}, \tag{5.1}$$

where **H** is the channel tap matrix given by $\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_{L_t} \end{bmatrix}^t$, and L_t is the number of transmit antennas. In this case, **H** is a $L_t \times m$ matrix and **X** is a $N \times m$ matrix; thus, $\mathbf{Y} = \begin{bmatrix} \boldsymbol{y}_1 & \boldsymbol{y}_2 & \cdots & \boldsymbol{y}_{L_t} \end{bmatrix}$ is a $N \times L_t$ matrix. The received noise measurement $\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_N \end{bmatrix}^t$, and the jamming interference measurement $\mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_N \end{bmatrix}^t$ are also $N \times L_t$ matrices. The proof of the received signal via Alamouti code is shown in the Appendix.

2. Complex De-spreading Procedure

Given the received signal **Y,** the de-spread vector is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\| \boldsymbol{c} \|}\mathbf{Y} = \mathbf{v}_S + \mathbf{n}_S + \mathbf{j}_S, \tag{5.2}$$

where $\mathbf{n}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}$ and $\mathbf{j}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}$ are the de-spread noise and interference matrices, and the de-spread signal $\mathbf{v}_S$ is given by

$$\mathbf{v}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X}\mathbf{H}^t = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{W}\mathbf{S}\mathbf{H}^t = \frac{\mathbf{W}\boldsymbol{C}^*\boldsymbol{c}}{\|\boldsymbol{c}\|^2}\mathbf{D}\mathbf{H}^t, \tag{5.3}$$

where $\mathbf{v}_S$ is a $N \times L_t$ matrix.

3. Symbol Recovery Procedure

The symbol recovery procedure is performed to extract the attenuated symbol. Again, we define the $1 \times N$ repeated Walsh vector $\widetilde{\boldsymbol{w}}$ and multiply it with the de-spread vector **V**. Thus,

$$\mathbf{Z} = \widetilde{\boldsymbol{w}}\mathbf{V} = \mathbf{z}_D + \mathbf{n}_D + \mathbf{j}_D, \tag{5.4}$$

where $\mathbf{n}_D = \widetilde{\boldsymbol{w}}\mathbf{n}_S$, $\mathbf{j}_D = \widetilde{\boldsymbol{w}}\mathbf{j}_S$, and

$$\mathbf{z}_D = \widetilde{\boldsymbol{w}}\mathbf{v}_S = \frac{\widetilde{\boldsymbol{w}}\mathbf{W}\boldsymbol{C}^*\boldsymbol{c}}{\|\boldsymbol{c}\|^2}\mathbf{D}\mathbf{H}^t = \mathbf{D}\mathbf{H}^t \tag{5.5}$$

is the recovered signal. Thus, the received vector **Z** is a $1 \times L_t$ matrix. It is clear that the desired symbol sequence **D** is available in $\mathbf{z}_D$, which is scaled by the channel tap matrix.

4. Detection via MRC

Finally, we can detect the symbols in **D** via MRC. We assume the channel matrix is known via channel estimation. The MRC operation takes the form $\mathbf{Z}\frac{\mathbf{H}^*}{\|\mathbf{h}\|}$, where $\mathbf{H}^*$ is the complex conjugate of the $m \times L_t$ channel tap matrix **H**, and the final received symbol vector in noise and jamming interference is given by

$$\mathbf{U} = \mathbf{Z}\frac{\mathbf{H}^*}{\|\mathbf{h}\|} = \mathbf{B} + \mathbf{n}_R + \mathbf{j}_R, \tag{5.6}$$

where $\mathbf{B} = \mathbf{z}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$, $\mathbf{n}_R = \mathbf{n}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$, and $\mathbf{j}_R = \mathbf{j}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$. Hence, the dimension of $\mathbf{U}$ is $1 \times m$, which matches the original transmitted symbol vector. Without any noise or interference, the recovered vector $\mathbf{U}$ is simply

$$\mathbf{B} = \mathbf{z}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|} = \mathbf{D}\mathbf{H}^t \frac{\mathbf{H}^*}{\|\mathbf{h}\|} = \mathbf{D}\|\mathbf{h}\|\mathbf{I}_{m\times m} = \|\mathbf{h}\|\mathbf{D}, \tag{5.7}$$

where **D**, which is scaled by ||**h**||, is available for detection.

C. NOISE AND INTERFERENCE POWER CALCULATIONS FOR FIXED DATA RATE CHANNEL

Recall that after symbol recovery, the received measurement is (5.4). We need to find the covariances of $\mathbf{n}_D = \tilde{\boldsymbol{w}}\mathbf{n}_S = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}$ and $\mathbf{j}_D = \tilde{\boldsymbol{w}}\mathbf{j}_S = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}$, where **J** is any of the three jamming interferences we consider: barrage, pulse-noise, and tone jamming. We also need to find the covariances of the MRC output $\mathbf{n}_R = \mathbf{n}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$, and $\mathbf{j}_R = \mathbf{j}_D \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$.

1. Receiver Noise

After the signal is received, the noise $N \times L_t$ matrix **N** is given by

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_N \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1L_t} \\ n_{21} & n_{22} & \cdots & n_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ n_{N1} & n_{N2} & \cdots & n_{NL_t} \end{bmatrix}. \tag{5.8}$$

If $\mathbf{n}_D = \tilde{\boldsymbol{w}}\mathbf{n}_S = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}$, then its covariance is given by

$$\mathrm{cov}\left(\mathbf{n}_D\right)=\mathrm{E}\left[\mathbf{n}_D^{\dagger}\mathbf{n}_D\right]=\mathrm{E}\left[\left(\widetilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}\right)^{\dagger}\left(\widetilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}\right)\right], \tag{5.9}$$

where † denotes the complex conjugate transpose. For notational convenience, we define the repeated $1\times N$ Walsh vector given in (3.12) and (3.13) as

$$\begin{aligned}\widetilde{\boldsymbol{w}}&=\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}\\ &=\begin{cases} \widetilde{\boldsymbol{w}}=\begin{bmatrix} \underbrace{w_1\cdots w_1}_{b}, & \underbrace{w_2\cdots w_2}_{b}, & \cdots, & \underbrace{w_M\cdots w_M}_{b} \end{bmatrix} & \textit{for method one}\\ \widetilde{\boldsymbol{w}}=\begin{bmatrix} \underbrace{\begin{bmatrix} w_1 & w_2\cdots w_M \end{bmatrix}}_{1}, & \cdots, & \underbrace{\begin{bmatrix} w_1 & w_2\cdots w_M \end{bmatrix}}_{b} \end{bmatrix} & \textit{for method two,}\end{cases}\end{aligned} \tag{5.10}$$

where $N = bM$. The de-spread noise vector is

$$\begin{aligned}\mathbf{n}_D=\widetilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}&=\frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0\\ 0 & c_2^* & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1L_t}\\ n_{21} & n_{22} & \cdots & n_{2L_t}\\ \vdots & \vdots & \ddots & \vdots\\ n_{N1} & n_{N2} & \cdots & n_{NL_t} \end{bmatrix}}_{N\times L_t}\\ &=\frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^*n_{11} & c_1^*n_{12} & \cdots & c_1^*n_{1L_t}\\ c_2^*n_{21} & c_2^*n_{22} & \cdots & c_2^*n_{2L_t}\\ \vdots & \vdots & \ddots & \vdots\\ c_N^*n_{N1} & c_N^*n_{N2} & \cdots & c_N^*n_{NL_t} \end{bmatrix}}_{N\times L_t}\\ &=\frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{N} w_a c_a^* n_{a1}\\ \sum_{a=1}^{N} w_a c_a^* n_{a2}\\ \vdots\\ \sum_{a=1}^{N} w_a c_a^* n_{aL_t} \end{bmatrix}^t}_{1\times L_t},\end{aligned} \tag{5.11}$$

where $\mathbf{n}_D$ is an $1\times L_t$ vector. Its Hermitian, which is a $L_t\times 1$ vector is given by

$$\mathbf{n}_D^{\dagger} = \left(\tilde{\boldsymbol{w}} \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|} \mathbf{N} \right)^{\dagger} = \mathbf{N}^{\dagger} \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|} \tilde{\boldsymbol{w}}^{\dagger}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} n_{11}^* & n_{21}^* & \cdots & n_{N1}^* \\ n_{12}^* & n_{22}^* & \cdots & n_{N2}^* \\ \vdots & \vdots & \ddots & \vdots \\ n_{1L_t}^* & n_{2L_t}^* & \cdots & n_{NL_t}^* \end{bmatrix}}_{L_t \times N} \underbrace{\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}}_{N \times N} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N \times 1}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} n_{11}^* c_1 & n_{21}^* c_2 & \cdots & n_{N1}^* c_N \\ n_{12}^* c_1 & n_{22}^* c_2 & \cdots & n_{N2}^* c_N \\ \vdots & \vdots & \ddots & \vdots \\ n_{1L_t}^* c_1 & n_{2L_t}^* c_2 & \cdots & n_{NL_t}^* c_N \end{bmatrix}}_{L_t \times N} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N \times 1} \tag{5.12}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \sum_{a=1}^{N} n_{a1}^* c_a w_a \\ \sum_{a=1}^{N} n_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{N} n_{aL_t}^* c_a w_a \end{bmatrix}}_{L_t \times 1}.$$

The covariance is

$$\operatorname{cov}\left(\mathbf{n}_D\right) = \mathrm{E}\left[\mathbf{n}_D^{\dagger} \mathbf{n}_D\right]$$

$$= \mathrm{E}\left[\frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \sum_{a=1}^{N} n_{a1}^* c_a w_a \\ \sum_{a=1}^{N} n_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{N} n_{aL_t}^* c_a w_a \end{bmatrix}}_{L_t \times 1} \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \sum_{a=1}^{N} w_a c_a^* n_{a1} \\ \sum_{a=1}^{N} w_a c_a^* n_{a2} \\ \vdots \\ \sum_{a=1}^{N} w_a c_a^* n_{aL_t} \end{bmatrix}^t}_{1 \times L_t} \right] \tag{5.13}$$

$$= \frac{1}{\|\boldsymbol{c}\|^2} \begin{bmatrix} \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{b1} n_{a1}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{bL_t} n_{a1}^* c_a w_a \right] \\ \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{b1} n_{a2}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{bL_t} n_{a2}^* c_a w_a \right] \\ \vdots & \ddots & \vdots \\ \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{b1} n_{aL_t}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* n_{bL_t} n_{aL_t}^* c_a w_a \right] \end{bmatrix}.$$

Since the sampled receiver noise is AWGN, we know that

$$\begin{aligned} \mathrm{E}\left[n_{ij} n_{kl}^{*} \right] &= \sigma^2, \quad i = k,\ j = l \\ &= 0, \qquad \textit{otherwise} \end{aligned} \tag{5.14}$$

and since $w_1^2 = w_2^2 = \cdots w_N^2 = 1$, $\sum_{a=1}^{N} \sum_{b=1}^{N} w_b c_b^{*} c_a w_a = \sum_{a=1}^{N} |\, c_a \,|^2$. Thus, the covariance (5.13) reduces to

$$\mathrm{cov}\left(\mathbf{n}_D \right) = \frac{1}{\| \boldsymbol{c} \|^2} \begin{bmatrix} \sum_{a=1}^{N} |\, c_a \,|^2 \sigma^2 & 0 & \cdots & 0 \\ 0 & \sum_{a=1}^{N} |\, c_a \,|^2 \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{a=1}^{N} |\, c_a \,|^2 \sigma^2 \end{bmatrix}. \tag{5.15}$$

Since, $\| \boldsymbol{c} \|^2 = \sum_{a=1}^{N} |\, c_a \,|^2$, the covariance is simply

$$\mathrm{cov}\left(\mathbf{n}_D \right) = \sigma_n^2 \mathrm{I}_{L_t \times L_t}. \tag{5.16}$$

The MRC noise vector is given by $\mathbf{n}_R = \mathbf{n}_D \dfrac{\mathbf{H}^{*}}{\| \mathbf{h} \|}$. Hence, the covariance of the MRC noise is given by

$$\begin{aligned} \mathrm{cov}\left(\mathbf{n}_R \right) = \mathrm{E}\left[\mathbf{n}_R^{\dagger} \mathbf{n}_R \right] &= \mathrm{E}\left[\left(\mathbf{n}_D \frac{\mathbf{H}^{*}}{\| \mathbf{h} \|} \right)^{\dagger} \left(\mathbf{n}_D \frac{\mathbf{H}^{*}}{\| \mathbf{h} \|} \right) \right] \\ &= \sigma_n^2 \frac{\mathbf{H}^{t}}{\| \mathbf{h} \|} \mathrm{I}_{L_t \times L_t} \frac{\mathbf{H}^{*}}{\| \mathbf{h} \|} \\ &= \sigma_n^2 \frac{\mathbf{H}^{t}}{\| \mathbf{h} \|} \frac{\mathbf{H}^{*}}{\| \mathbf{h} \|} = \sigma_n^2 \frac{\mathbf{H}^{\dagger} \mathbf{H}}{\| \mathbf{h} \|^2} \\ &= \sigma_n^2 \frac{\| \mathbf{h} \|^2}{\| \mathbf{h} \|^2} \mathrm{I}_{m \times m} = \sigma_n^2 \mathrm{I}_{m \times m}. \end{aligned} \tag{5.17}$$

2. Barrage Jamming

Here we consider the jamming due to barrage noise, which is modeled as broadband noise, i.e., AWGN. For an un-spread system, the jamming sample is

distributed according to $\mathcal{CN}\left(0,\sigma_B^2\right)$, where the notation stands for complex AWGN with zero mean and variance σ_B^2. Let the variance of the complex noise sample be $\sigma_{B'}^2$. Thus, for a spread spectrum system, the power of the noise is decreased by the spreading gain *N*. The complex interference sample is then distributed according to $\mathcal{CN}\left(0,\sigma_{B'}^2=\frac{\sigma_B^2}{N}\right)$.

After the signal is received, the barrage jamming $N\times L_t$ matrix is given as

$$\mathbf{J}_{\mathrm{B}}=\begin{bmatrix}\mathbf{j}_1\\ \mathbf{j}_2\\ \vdots\\ \mathbf{j}_N\end{bmatrix}=\begin{bmatrix}\varphi_{11} & \varphi_{12} & \cdots & \varphi_{1L}\\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2L}\\ \vdots & \vdots & \ddots & \vdots\\ \varphi_{N1} & \varphi_{N2} & \cdots & \varphi_{NL}\end{bmatrix}. \tag{5.18}$$

If $\mathbf{j}_B=\tilde{\boldsymbol{w}}\mathbf{j}_{\mathrm{S}}=\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{B}}$, then its covariance is given by

$$\mathrm{cov}\left(\mathbf{j}_B\right)=\mathrm{E}\left[\mathbf{j}_B^{\dagger}\mathbf{j}_B\right]=\mathrm{E}\left[\left(\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{B}}\right)^{\dagger}\left(\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{B}}\right)\right]. \tag{5.19}$$

The de-spread jamming vector is

$$\mathbf{j}_B = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{B}} = \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1L_t} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{N1} & \varphi_{N2} & \cdots & \varphi_{NL_t} \end{bmatrix}}_{N\times L_t}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^*\varphi_{11} & c_1^*\varphi_{12} & \cdots & c_1^*\varphi_{1L_t} \\ c_2^*\varphi_{21} & c_2^*\varphi_{22} & \cdots & c_2^*\varphi_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_N^*\varphi_{N1} & c_N^*\varphi_{N2} & \cdots & c_N^*\varphi_{NL_t} \end{bmatrix}}_{N\times L_t} \tag{5.20}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{N} w_a c_a^*\varphi_{a1} \\ \sum_{a=1}^{N} w_a c_a^*\varphi_{a2} \\ \vdots \\ \sum_{a=1}^{N} w_a c_a^*\varphi_{aL_t} \end{bmatrix}^t}_{1\times L_t},$$

and its Hermitian is given by

$$\mathbf{j}_B^{\dagger} = \left(\tilde{\boldsymbol{w}} \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|} \mathbf{J}_{\mathrm{B}} \right)^{\dagger} = \mathbf{J}_{\mathrm{B}}^{\dagger} \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|} \tilde{\boldsymbol{w}}^{\dagger}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \varphi_{11}^* & \varphi_{21}^* & \cdots & \varphi_{N1}^* \\ \varphi_{12}^* & \varphi_{22}^* & \cdots & \varphi_{N2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{1L_t}^* & \varphi_{2L_t}^* & \cdots & \varphi_{NL_t}^* \end{bmatrix}}_{L_t \times N} \underbrace{\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}}_{N \times N} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N \times 1}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \varphi_{11}^* c_1 & \varphi_{21}^* c_2 & \cdots & \varphi_{N1}^* c_N \\ \varphi_{12}^* c_1 & \varphi_{22}^* c_2 & \cdots & \varphi_{N2}^* c_N \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{1L_t}^* c_1 & \varphi_{2L_t}^* c_2 & \cdots & \varphi_{NL_t}^* c_N \end{bmatrix}}_{L_t \times N} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N \times 1} \tag{5.21}$$

$$= \frac{1}{\|\boldsymbol{c}\|} \underbrace{\begin{bmatrix} \sum_{a=1}^{N} \varphi_{a1}^* c_a w_a \\ \sum_{a=1}^{N} \varphi_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{N} \varphi_{aL_t}^* c_a w_a \end{bmatrix}}_{L_t \times 1}.$$

The covariance is

$$\mathrm{cov}\left(\mathbf{j}_B\right) = \mathrm{E}\left[\mathbf{j}_B^{\dagger} \mathbf{j}_B\right]$$

$$= \mathrm{E}\left[\underbrace{\frac{1}{\|\boldsymbol{c}\|} \begin{bmatrix} \sum_{a=1}^{N} \varphi_{a1}^* c_a w_a \\ \sum_{a=1}^{N} \varphi_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{N} \varphi_{aL_t}^* c_a w_a \end{bmatrix}}_{L_t \times 1} \underbrace{\frac{1}{\|\boldsymbol{c}\|} \begin{bmatrix} \sum_{a=1}^{N} w_a c_a^* \varphi_{a1} \\ \sum_{a=1}^{N} w_a c_a^* \varphi_{a2} \\ \vdots \\ \sum_{a=1}^{N} w_a c_a^* \varphi_{aL_t} \end{bmatrix}^t}_{1 \times L_t} \right] \tag{5.22}$$

$$= \frac{1}{\|\boldsymbol{c}\|^2} \begin{bmatrix} \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{b1} \varphi_{a1}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{bL_t} \varphi_{a1}^* c_a w_a \right] \\ \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{b1} \varphi_{a2}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{bL_t} \varphi_{a2}^* c_a w_a \right] \\ \vdots & \ddots & \vdots \\ \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{b1} \varphi_{aL_t}^* c_a w_a \right] & \cdots & \sum_{a=1}^{N} \sum_{b=1}^{N} \mathrm{E}\left[w_b c_b^* \varphi_{bL_t} \varphi_{aL_t}^* c_a w_a \right] \end{bmatrix}.$$

Since the complex noise sample is AWGN, we know that

$$\mathrm{E}\left[\varphi_{ij}\varphi_{kl}^{*}\right]=\frac{\sigma_B^2}{N},\quad i=k,\ j=l \\ =0,\qquad otherwise \tag{5.23}$$

and since $w_1^2=w_2^2=\cdots w_N^2=1$, $\sum_{a=1}^{N}\sum_{b=1}^{N}w_b c_b^{*}c_a w_a=\sum_{a=1}^{N}|c_a|^2$. Thus, the covariance (5.22) reduces to

$$\mathrm{cov}\left(\mathbf{j}_B\right)=\frac{1}{\|\boldsymbol{c}\|^2}\begin{bmatrix}\sum_{a=1}^{N}|c_a|^2\frac{\sigma_B^2}{N} & 0 & \cdots & 0\\ 0 & \sum_{a=1}^{N}|c_a|^2\frac{\sigma_B^2}{N} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sum_{a=1}^{N}|c_a|^2\frac{\sigma_B^2}{N}\end{bmatrix}, \tag{5.24}$$

and since $\|\boldsymbol{c}\|^2=\sum_{a=1}^{N}|c_a|^2$, the covariance is simply

$$\mathrm{cov}\left(\mathbf{j}_B\right)=\frac{\sigma_B^2}{N}\mathbf{I}_{L_t\times L_t}. \tag{5.25}$$

The MRC interference vector is given by $\mathbf{j}_R=\mathbf{j}_B\frac{\mathbf{H}^{*}}{\|\mathbf{h}\|}$. Hence, the covariance of the MRC noise is given by

$$\begin{aligned}\mathrm{cov}\left(\mathbf{j}_R\right)=\mathrm{E}\left[\mathbf{j}_R^{\dagger}\mathbf{j}_R\right]&=\mathrm{E}\left[\left(\mathbf{j}_B\frac{\mathbf{H}^{*}}{\|\mathbf{h}\|}\right)^{\dagger}\left(\mathbf{j}_B\frac{\mathbf{H}^{*}}{\|\mathbf{h}\|}\right)\right]\\ &=\frac{\sigma_B^2}{N}\frac{\mathbf{H}^{t}}{\|\mathbf{h}\|}\mathbf{I}_{L_t\times L_t}\frac{\mathbf{H}^{*}}{\|\mathbf{h}\|}\\ &=\frac{\sigma_B^2}{N}\frac{\mathbf{H}^{t}}{\|\mathbf{h}\|}\frac{\mathbf{H}^{*}}{\|\mathbf{h}\|}=\frac{\sigma_B^2}{N}\frac{\mathbf{H}^{\dagger}\mathbf{H}}{\|\mathbf{h}\|^2}\\ &=\frac{\sigma_B^2}{N}\frac{\|\mathbf{h}\|^2}{\|\mathbf{h}\|^2}\mathbf{I}_{m\times m}=\frac{\sigma_B^2}{N}\mathbf{I}_{m\times m}.\end{aligned} \tag{5.26}$$

3. Pulsed-Noise Jamming

Here we consider pulsed-noise jamming, which is broadband noise received in periodic spurts. The period or duty cycle is determined by ρ (between 0 and 1), which is the percentage of time the jammer is on. When $\rho = 0$ the jammer is consider off, and when $\rho = 1$, it is consider barrage jamming. For a spread spectrum system, the complex interference sample is distributed according to $\mathcal{CN}\left(0, \frac{\sigma_P^2}{\rho N}\right)$. After the signal is received, the pulsed-noise jamming $N \times L_t$ matrix is given as

$$\mathbf{J}_{\mathrm{P}} = \begin{bmatrix} \mathbf{j}_1 \\ \mathbf{j}_2 \\ \vdots \\ \mathbf{j}_N \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1L_t} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N1} & \psi_{N2} & \cdots & \psi_{NL_t} \end{bmatrix}. \tag{5.27}$$

If $\mathbf{j}_P = \tilde{\boldsymbol{w}}\mathbf{j}_{\mathrm{S}} = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{P}}$, then its covariance is given by

$$\mathrm{cov}\left(\mathbf{j}_P\right) = \mathrm{E}\left[\mathbf{j}_P^{\dagger}\mathbf{j}_P\right] = \mathrm{E}\left[\left(\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{P}}\right)^{\dagger}\left(\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{P}}\right)\right]. \tag{5.28}$$

The de-spread jamming vector is

$$\mathbf{j}_P = \tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{P}} = \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1L_t} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N1} & \psi_{N2} & \cdots & \psi_{NL_t} \end{bmatrix}}_{N\times L_t}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1\times N}\underbrace{\begin{bmatrix} c_1^*\psi_{11} & c_1^*\psi_{12} & \cdots & c_1^*\psi_{1L_t} \\ c_2^*\psi_{21} & c_2^*\psi_{22} & \cdots & c_2^*\psi_{2L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_N^*\psi_{N1} & c_N^*\psi_{N2} & \cdots & c_N^*\psi_{NL_t} \end{bmatrix}}_{N\times L_t} \tag{5.29}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{N} w_a c_a^* \psi_{a1} \\ \sum_{a=1}^{N} w_a c_a^* \psi_{a2} \\ \vdots \\ \sum_{a=1}^{N} w_a c_a^* \psi_{aL_t} \end{bmatrix}^t}_{1\times L_t},$$

and its Hermitian is given by

$$\mathbf{j}_P^\dagger = \left(\tilde{\boldsymbol{w}}\frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}_{\mathrm{P}}\right)^\dagger = \mathbf{J}_{\mathrm{P}}^\dagger\frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\tilde{\boldsymbol{w}}^\dagger$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \psi_{11}^* & \psi_{21}^* & \cdots & \psi_{N1}^* \\ \psi_{12}^* & \psi_{22}^* & \cdots & \psi_{N2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{1L_t}^* & \psi_{2L_t}^* & \cdots & \psi_{NL_t}^* \end{bmatrix}}_{L_t\times N}\underbrace{\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N \end{bmatrix}}_{N\times N}\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N\times 1}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \psi_{11}^* c_1 & \psi_{21}^* c_2 & \cdots & \psi_{N1}^* c_N \\ \psi_{12}^* c_1 & \psi_{22}^* c_2 & \cdots & \psi_{N2}^* c_N \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{1L_t}^* c_1 & \psi_{2L_t}^* c_2 & \cdots & \psi_{NL_t}^* c_N \end{bmatrix}}_{L_t\times N}\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}_{N\times 1} \tag{5.30}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{N} \psi_{a1}^* c_a w_a \\ \sum_{a=1}^{N} \psi_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{N} \psi_{aL_t}^* c_a w_a \end{bmatrix}}_{L_t\times 1}.$$

The covariance is

$$\begin{aligned}\operatorname{cov}\left(\mathbf{j}_P\right) &= \mathrm{E}\left[\mathbf{j}_P^{\dagger}\mathbf{j}_P\right] \\ &= \mathrm{E}\left[\frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\sum_{a=1}^{N}\psi_{a1}^{*}c_a w_a \\ \sum_{a=1}^{N}\psi_{a2}^{*}c_a w_a \\ \vdots \\ \sum_{a=1}^{N}\psi_{aL_t}^{*}c_a w_a\end{bmatrix}}_{L_t\times 1}\frac{1}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\sum_{a=1}^{N}w_a c_a^{*}\psi_{a1} \\ \sum_{a=1}^{N}w_a c_a^{*}\psi_{a2} \\ \vdots \\ \sum_{a=1}^{N}w_a c_a^{*}\psi_{aL_t}\end{bmatrix}^{t}}_{1\times L_t}\right] \\ &= \frac{1}{\|\boldsymbol{c}\|^2}\begin{bmatrix}\sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{b1}\psi_{a1}^{*}c_a w_a\right] & \cdots & \sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{bL_t}\psi_{a1}^{*}c_a w_a\right] \\ \sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{b1}\psi_{a2}^{*}c_a w_a\right] & \cdots & \sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{bL_t}\psi_{a2}^{*}c_a w_a\right] \\ \vdots & \ddots & \vdots \\ \sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{b1}\psi_{aL_t}^{*}c_a w_a\right] & \cdots & \sum_{a=1}^{N}\sum_{b=1}^{N}\mathrm{E}\left[w_b c_b^{*}\psi_{bL_t}\psi_{aL_t}^{*}c_a w_a\right]\end{bmatrix}.\end{aligned} \tag{5.31}$$

Since the complex noise sample is AWGN, we know that

$$\begin{aligned}\mathrm{E}\left[\psi_{ij}\psi_{kl}^{*}\right] &= \frac{\sigma_P^2}{\rho N}, \quad i=k,\ j=l \\ &= 0, \qquad \textit{otherwise}\end{aligned} \tag{5.32}$$

and since $w_1^2 = w_2^2 = \cdots w_N^2 = 1$, $\sum_{a=1}^{N}\sum_{b=1}^{N} w_b c_b^{*} c_a w_a = \sum_{a=1}^{N}|c_a|^2$. Thus, the covariance (5.31) reduces to

$$\operatorname{cov}\left(\mathbf{j}_P\right) = \frac{1}{\|\boldsymbol{c}\|^2}\begin{bmatrix}\sum_{a=1}^{N}|c_a|^2\frac{\sigma_P^2}{\rho N} & 0 & \cdots & 0 \\ 0 & \sum_{a=1}^{N}|c_a|^2\frac{\sigma_P^2}{\rho N} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{a=1}^{N}|c_a|^2\frac{\sigma_P^2}{\rho N}\end{bmatrix}, \tag{5.33}$$

and since $\|\boldsymbol{c}\|^2 = \sum_{a=1}^{N}|c_a|^2$, the covariance is simply

$$\operatorname{cov}\left(\mathbf{j}_P\right) = \frac{\sigma_P^2}{\rho N}\mathrm{I}_{L_t\times L_t}. \tag{5.34}$$

The MRC interference vector is given by $\mathbf{j}_S = \mathbf{j}_P \frac{\mathbf{H}^*}{\|\mathbf{h}\|}$. Hence, the covariance of the MRC noise is given by

$$\begin{aligned}\operatorname{cov}(\mathbf{j}_S) = \mathrm{E}\left[\mathbf{j}_S^\dagger \mathbf{j}_S\right] &= \mathrm{E}\left[\left(\mathbf{j}_P \frac{\mathbf{H}^*}{\|\mathbf{h}\|}\right)^\dagger \left(\mathbf{j}_P \frac{\mathbf{H}^*}{\|\mathbf{h}\|}\right)\right] \\ &= \frac{\sigma_P^2}{\rho N} \frac{\mathbf{H}^t}{\|\mathbf{h}\|} \mathrm{I}_{L_t \times L_t} \frac{\mathbf{H}^*}{\|\mathbf{h}\|} \\ &= \frac{\sigma_P^2}{\rho N} \frac{\mathbf{H}^t}{\|\mathbf{h}\|} \frac{\mathbf{H}^*}{\|\mathbf{h}\|} = \frac{\sigma_P^2}{\rho N} \frac{\mathbf{H}^\dagger \mathbf{H}}{\|\mathbf{h}\|^2} \\ &= \frac{\sigma_P^2}{\rho N} \frac{\|\mathbf{h}\|^2}{\|\mathbf{h}\|^2} \mathrm{I}_{m \times m} = \frac{\sigma_P^2}{\rho N} \mathrm{I}_{m \times m}.\end{aligned} \tag{5.35}$$

4. Tone Jamming

In tone jamming, the interference is caused by an interference tone at the carrier frequency. This results in a phase offset when the passband signal is downconverted to baseband. Recall from (5.1) that the interference prior to de-spreading is $\mathbf{J} = \begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_N \end{bmatrix}^t$, and an arbitrary jamming interference $\mathbf{j}$ is given by $\mathbf{j} = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_{L_t} \end{bmatrix} = \frac{A_J}{2} \begin{bmatrix} e^{-j\theta_1} & e^{-j\theta_2} & \cdots & e^{-j\theta_{L_t}} \end{bmatrix}$, where A_J is the amplitude of the carrier tone. After the signal is received, the tone jamming $N \times L_t$ matrix is given as

$$\mathbf{J}_\mathrm{T} = \frac{A_J}{2} \begin{bmatrix} e^{-j\theta_{11}} & e^{-j\theta_{21}} & \cdots & e^{-j\theta_{L_t 1}} \\ e^{-j\theta_{12}} & e^{-j\theta_{22}} & \cdots & e^{-j\theta_{L_t 1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\theta_{1N}} & e^{-j\theta_{2N}} & \cdots & e^{-j\theta_{L_t N}} \end{bmatrix}, \tag{5.36}$$

and the de-spread jamming vector is given by

$$\mathbf{j}_Q = \tilde{\boldsymbol{w}} \frac{\boldsymbol{C}^*}{\| \boldsymbol{c} \|} \mathbf{J}_{\mathrm{T}}$$

$$= \frac{A_J}{2 \| \boldsymbol{c} \|} \underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}}_{1 \times N} \underbrace{\begin{bmatrix} c_1^* & 0 & 0 & 0 \\ 0 & c_2^* & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c_N^* \end{bmatrix}}_{N \times N} \underbrace{\begin{bmatrix} e^{-j\theta_{11}} & e^{-j\theta_{21}} & \cdots & e^{-j\theta_{L_t 1}} \\ e^{-j\theta_{12}} & e^{-j\theta_{22}} & \cdots & e^{-j\theta_{L_t 1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\theta_{1N}} & e^{-j\theta_{2N}} & \cdots & e^{-j\theta_{L_t N}} \end{bmatrix}}_{N \times L_t} \tag{5.37}$$

$$= \frac{A_J}{2 \| \boldsymbol{c} \|} \begin{bmatrix} w_1 c_1^* e^{-j\theta_{11}} + w_2 c_2^* e^{-j\theta_{12}} + \cdots + w_N c_N^* e^{-j\theta_{1N}} \\ w_1 c_1^* e^{-j\theta_{21}} + w_2 c_2^* e^{-j\theta_{22}} + \cdots + w_N c_N^* e^{-j\theta_{2N}} \\ \vdots \\ w_1 c_1^* e^{-j\theta_{L_t 1}} + w_2 c_2^* e^{-j\theta_{L_t 2}} + \cdots + w_N c_N^* e^{-j\theta_{L_t N}} \end{bmatrix}^t.$$

It can be shown that the tone vector is given by

$$\mathbf{j}_Q = \frac{A_J}{2 \| \boldsymbol{c} \|} \begin{bmatrix} \sum_{a=1}^N w_a \left(c_{Ia}^* - j c_{Qa}^* \right) \left(\cos\theta_{1a} - j \sin\theta_{1a} \right) \\ \sum_{a=1}^N w_a \left(c_{Ia}^* - j c_{Qa}^* \right) \left(\cos\theta_{2a} - j \sin\theta_{2a} \right) \\ \vdots \\ \sum_{a=1}^N w_a \left(c_{Ia}^* - j c_{Qa}^* \right) \left(\cos\theta_{La} - j \sin\theta_{L_t a} \right) \end{bmatrix}^t$$

$$= \frac{A_J}{2 \| \boldsymbol{c} \|} \begin{bmatrix} \sum_{a=1}^N w_a \left(c_{Ia} \cos\theta_{1a} - c_{Qa} \sin\theta_{1a} - j c_{Qa} \cos\theta_{1a} - j c_{Ia} \sin\theta_{1a} \right) \\ \sum_{a=1}^N w_a \left(c_{Ia} \cos\theta_{2a} - c_{Qa} \sin\theta_{2a} - j c_{Qa} \cos\theta_{2a} - j c_{Ia} \sin\theta_{2a} \right) \\ \vdots \\ \sum_{a=1}^N w_a \left(c_{Ia} \cos\theta_{L_t a} - c_{Qa} \sin\theta_{L_t a} - j c_{Qa} \cos\theta_{L_t a} - j c_{Ia} \sin\theta_{L_t a} \right) \end{bmatrix}^t. \tag{5.38}$$

Note that $c_{Ia}^* = c_{Ia}$ and $c_{Qa}^* = c_{Qa}$. Since the chips are real constants of 1 and -1, $\tilde{\boldsymbol{w}}$ can be all 1s or an even number of 1s and -1s. We assume that there is an even number of chips in the PN code since the Walsh sequences are always even. Furthermore, assume that the number of 1s outnumbers -1s by α such that $\sum_{a=1}^N w_a c_{Ia} = \sum_{a=1}^N w_a c_{Qa} = \alpha$. Then (5.38) can be simplified to

$$\mathbf{j}_Q = \left(\frac{A_J}{2\|\boldsymbol{c}\|}\right)(\alpha)\begin{bmatrix} \left(\cos\theta_1 - j\sin\theta_1\right) - \left(j\cos\theta_1 + \sin\theta_1\right) \\ \left(\cos\theta_2 - j\sin\theta_2\right) - \left(j\cos\theta_2 + \sin\theta_2\right) \\ \vdots \\ \left(\cos\theta_{L_t} - j\sin\theta_{L_t}\right) - \left(j\cos\theta_{L_t} + \sin\theta_{L_t}\right) \end{bmatrix}^t \tag{5.39}$$

$$= \frac{\alpha A_J}{2\|\boldsymbol{c}\|}\begin{bmatrix} e^{-j\theta_1} - je^{-j\theta_1} \\ e^{-j\theta_2} - je^{-j\theta_2} \\ \vdots \\ e^{-j\theta_{L_t}} - je^{-j\theta_{L_t}} \end{bmatrix}^t.$$

Note that if $\alpha = 0$, the tone jamming is completely nulled out. Thus, the covariance is given by

$$\begin{aligned} \mathrm{cov}\left(\mathbf{j}_Q\right) &= \mathrm{E}\left[\left(\mathbf{j}_Q\right)^{\dagger}\mathbf{j}_Q\right] \\ &= \mathrm{E}\left[\frac{\alpha A_J}{2\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} e^{j\theta_1} + je^{j\theta_1} \\ e^{j\theta_2} + je^{j\theta_2} \\ \vdots \\ e^{j\theta_{L_t}} + je^{j\theta_{L_t}} \end{bmatrix}}_{L_t\times 1}\frac{\alpha A_J}{2\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} e^{-j\theta_1} - je^{-j\theta_1} \\ e^{-j\theta_2} - je^{-j\theta_2} \\ \vdots \\ e^{-j\theta_{L_t}} - je^{-j\theta_{L_t}} \end{bmatrix}^t}_{1\times L_t}\right] \\ &= 2\left(\frac{\alpha A_J}{2\|\boldsymbol{c}\|}\right)^2\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{L_t\times L_t} \\ &= \frac{\alpha^2 A_J^2}{2\|\boldsymbol{c}\|^2}\mathrm{I}_{L_t\times L_t} = \frac{\alpha^2 A_J^2}{4N}\mathrm{I}_{L_t\times L_t}, \end{aligned} \tag{5.40}$$

where $\mathrm{I}_{L_t\times L_t}$ is an identity matrix, and $\|\boldsymbol{c}\|^2 = 2N$. The MRC interference vector is given by $\mathbf{j}_T = \mathbf{j}_Q\dfrac{\mathbf{H}^*}{\|\mathbf{h}\|}$. Hence, the covariance of the MRC noise is given by

$$\begin{aligned}\operatorname{cov}\left(\mathbf{j}_T\right)=\mathrm{E}\left[\mathbf{j}_T^{\dagger}\mathbf{j}_T\right]&=\mathrm{E}\left[\left(\mathbf{j}_Q\frac{\mathbf{H}^*}{\|\mathbf{h}\|}\right)^{\dagger}\left(\mathbf{j}_Q\frac{\mathbf{H}^*}{\|\mathbf{h}\|}\right)\right]\\&=\frac{\alpha^2A_J^2}{4N}\frac{\mathbf{H}^t}{\|\mathbf{h}\|}\mathbf{I}_{L_t\times L_t}\frac{\mathbf{H}^*}{\|\mathbf{h}\|}\\&=\frac{\alpha^2A_J^2}{4N}\frac{\mathbf{H}^t}{\|\mathbf{h}\|}\frac{\mathbf{H}^*}{\|\mathbf{h}\|}=\frac{\alpha^2A_J^2}{4N}\frac{\mathbf{H}^{\dagger}\mathbf{H}}{\|\mathbf{h}\|^2}\\&=\frac{\alpha^2A_J^2}{4N}\frac{\|\mathbf{h}\|^2}{\|\mathbf{h}\|^2}\mathbf{I}_{m\times m}=\frac{\alpha^2A_J^2}{4N}\mathbf{I}_{m\times m}.\end{aligned}\tag{5.41}$$

D. INTEGRATED SPREADING WITH OSTBC SIGNALING IN FADING, NOISE AND JAMMING FOR MULTI-RATE CHANNELS

Here, we include the OSTBC to the noise and interference analyses of multi-rate channels in Chapter IV.

1. Received Signal with Noise and Interference

Here we extend the procedure in (4.9), where the $N\times m_1$ integrated spread matrix $\mathbf{X}$ is given as $\mathbf{X}=\mathbf{X}_1+\mathbf{X}_2=\frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\left(\tilde{\mathbf{S}}^{(1)}+\tilde{\mathbf{S}}^{(2)}\right)$. When the signal goes through an AWGN channel and interference, the received signal is given by

$$\mathbf{Y}=\mathbf{X}\mathbf{H}^t+\mathbf{N}+\mathbf{J},\tag{5.42}$$

where $\mathbf{H}$ is the channel tap matrix is given by $\mathbf{H}=\begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_{L_t}\end{bmatrix}$, and L_t is the number of transmit antennas. In this case, $\mathbf{H}$ is a $L_t\times m_1$ matrix and $\mathbf{X}=\begin{bmatrix}\mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{m_1}\end{bmatrix}$ is a $N\times m_1$ matrix; thus, $\mathbf{Y}=\begin{bmatrix}\boldsymbol{y}_1 & \boldsymbol{y}_2 & \cdots & \boldsymbol{y}_{L_t}\end{bmatrix}$ is a $N\times L_t$ matrix. The received noise measurement $\mathbf{N}=\begin{bmatrix}\mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_N\end{bmatrix}^t$, and the jamming interference measurement $\mathbf{J}=\begin{bmatrix}\mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_N\end{bmatrix}^t$ are also $N\times L_t$ matrices.

2. Complex De-spreading Procedure

Given received signal **Y,** the de-spread vector is given by

$$\mathbf{V} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{Y} = \mathbf{v}_S + \mathbf{n}_S + \mathbf{j}_S, \tag{5.43}$$

where $\mathbf{n}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{N}$ and $\mathbf{j}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{J}$ are the "de-spread" noise and interference matrices, and the de-spread received signal $\mathbf{v}_S$ (without noise and interference) is given by

$$\mathbf{v}_S = \frac{\boldsymbol{C}^*}{\|\boldsymbol{c}\|}\mathbf{X}\mathbf{H}^t = \frac{\boldsymbol{C}^*\boldsymbol{C}}{\|\boldsymbol{c}\|^2}\left(\tilde{\mathbf{S}}^{(1)} + \hat{\mathbf{S}}^{(2)}\right)\mathbf{H}^t, \tag{5.44}$$

where $\mathbf{v}_S$ is a $N \times L_t$ matrix. The proof of the received signal for channel two via an Alamouti code is shown in the Appendix.

3. Symbol Recovery Procedure

The symbol recovery procedure is performed to extract the attenuated symbol. For channel one, we use the $1 \times N$ "repeated" Walsh vector $\tilde{\boldsymbol{w}}^{(1)}$ given in (4.12) and multiply it with the de-spread vector **V**. Thus,

$$\mathbf{Z}^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{V} = \mathbf{z}_D^{(1)} + \mathbf{n}_D^{(1)} + \mathbf{j}_D^{(1)}, \tag{5.45}$$

where $\mathbf{n}_D^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{n}_S$, $\mathbf{j}_D^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{j}_S$. Using (4.16), we have the received attenuated signal without noise and interference in the form

$$\mathbf{z}_D^{(1)} = \tilde{\boldsymbol{w}}^{(1)}\mathbf{v}_S = \frac{\tilde{\boldsymbol{w}}^{(1)}}{\|\boldsymbol{c}\|^2}\boldsymbol{C}^*\boldsymbol{C}\left(\tilde{\mathbf{S}}^{(2)} + \hat{\mathbf{S}}^{(2)}\right)\left(\mathbf{H}^{(1)}\right)^t = \mathbf{D}^{(1)}\left(\mathbf{H}^{(1)}\right)^t. \tag{5.46}$$

The channel tap matrix $\mathbf{H}^{(1)}$ is a $L_t \times m_1$ matrix. Thus, the received signal $\mathbf{Z}^{(1)}$ is a $1 \times L_t$ matrix. It is clear that the desired symbol vector $\mathbf{D}^{(1)}$ is available in $\mathbf{z}_D^{(1)}$, which is scaled by the channel tap matrix.

For channel two, we assume that the mapping of $\mathbf{V} \to \widehat{\mathbf{V}}$ is performed in (4.17), where $\widehat{\mathbf{V}}$ is now a $(N/2)\times L_t$ matrix given by

$$\widehat{\mathbf{V}} = \widehat{\mathbf{v}}_S + \widehat{\mathbf{n}}_S + \widehat{\mathbf{j}}_S, \tag{5.47}$$

where $\widehat{\mathbf{n}}_S$ and $\widehat{\mathbf{j}}_S$ are the mapped $(N/2)\times 2L_t$ matrices. Now, we use the $1\times(N/2)$ repeated Walsh vector $\widetilde{\boldsymbol{w}}^{(2)}$ given in (4.19). The recovered attenuated signal is given by

$$\mathbf{Z}^{(2)} = 2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{V}} = \mathbf{z}_D^{(2)} + \mathbf{n}_D^{(2)} + \mathbf{j}_D^{(2)}, \tag{5.48}$$

where $\mathbf{n}_D^{(2)} = 2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{n}}_S$, $\mathbf{j}_D^{(2)} = 2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_S$, and

$$\mathbf{z}_D^{(2)} = 2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{v}}_S = 2\widetilde{\boldsymbol{w}}^{(2)}\left(\widehat{\mathbf{S}}^{(1)} + \widehat{\mathbf{S}}^{(2)}\right)^t\left(\mathbf{H}^{(2)}\right)^t = \mathbf{D}^{(2)}\left(\mathbf{H}^{(2)}\right)^t \tag{5.49}$$

is the recovered attenuated signal without noise and interference. The factor two is the normalization factor. Note that $\widehat{\mathbf{v}}_S$ is the de-spread $(N/2)\times 2L_t$ matrix without noise and interference. The received signal $\mathbf{Z}^{(2)}$ is a $1\times 2L_t$ matrix, and the desired symbol vector $\mathbf{D}^{(2)}$ is available in $\mathbf{z}_D^{(2)}$, which is scaled by the channel tap matrix. The channel tap matrix $\mathbf{H}^{(2)}$ in this case is the mapped $2L_t \times m_2$ matrix. For example, using $\mathbf{D}^{(2)}$ in *example 3*, where $\mathbf{D}^{(2)} = \begin{bmatrix} D_1^{(2)} & D_2^{(2)} & D_3^{(2)} & D_4^{(2)} \end{bmatrix}$, the channel tap matrix $\mathbf{H}^{(2)}$ is

$$\mathbf{H}^{(2)} = \underbrace{\begin{bmatrix} h_1 & h_2 & 0 & 0 \\ h_2^* & -h_1^* & 0 & 0 \\ 0 & 0 & h_1 & h_2 \\ 0 & 0 & h_2^* & -h_1^* \end{bmatrix}}_{2L_t \times m_2}. \tag{5.50}$$

The recovered attenuated signal without noise and interference is given by

$$\mathbf{z}_D^{(2)} = \mathbf{D}^{(2)}\left(\mathbf{H}^{(2)}\right)^t = \underbrace{\begin{bmatrix} D_1^{(2)} & D_2^{(2)} & D_3^{(2)} & D_4^{(2)} \end{bmatrix}}_{1\times m_2} \underbrace{\begin{bmatrix} h_1 & h_2^* & 0 & 0 \\ h_2 & -h_1^* & 0 & 0 \\ 0 & 0 & h_1 & h_2^* \\ 0 & 0 & h_2 & -h_1^* \end{bmatrix}}_{m_2\times 2L_t}$$

$$= \underbrace{\begin{bmatrix} h_1 D_1^{(2)} + h_2 D_2^{(2)} & h_2^* D_1^{(2)} - h_1^* D_2^{(2)} & h_1 D_3^{(2)} + h_2 D_4^{(2)} & h_2^* D_3^{(2)} - h_1^* D_4^{(2)} \end{bmatrix}}_{1\times 2L_t}. \quad (5.51)$$

4. Detection via MRC

To detect the symbols in $\mathbf{D}^{(1)}$, the MRC operation takes the form $\mathbf{Z}^{(1)}\dfrac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|}$, where $\left(\mathbf{H}^{(1)}\right)^*$ is the complex conjugate of the $m_1 \times L_t$ channel tap matrix $\mathbf{H}^{(1)}$, and the final received symbols in noise and jamming interference are given by

$$\mathbf{U}^{(1)} = \mathbf{Z}^{(1)}\frac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|} = \mathbf{B}^{(1)} + \mathbf{n}_R^{(1)} + \mathbf{j}_R^{(1)}, \quad (5.52)$$

where $\mathbf{B}^{(1)} = \mathbf{z}_D^{(1)}\dfrac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|}$, $\mathbf{n}_R^{(1)} = \mathbf{n}_D^{(1)}\dfrac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|}$, and $\mathbf{j}_R^{(1)} = \mathbf{j}_D^{(1)}\dfrac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|}$. Now, the dimension of $\mathbf{U}^{(1)}$ is $1\times m_1$, which matches the original symbols sent in channel one. Without any noise and interference, the recovered vector $\mathbf{U}^{(1)}$ is

$$\mathbf{B}^{(1)} = \mathbf{z}_D^{(1)}\frac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|} = \mathbf{D}^{(1)}\left(\mathbf{H}^{(1)}\right)^t\frac{\left(\mathbf{H}^{(1)}\right)^*}{\|\mathbf{h}^{(1)}\|} = \mathbf{D}^{(1)}\|\mathbf{h}^{(1)}\|\mathbf{I}_{m_1\times m_1} = \|\mathbf{h}^{(1)}\|\mathbf{D}^{(1)}, \quad (5.53)$$

where $\mathbf{D}^{(1)}$, which is scaled by $\|\mathbf{h}\|$ is available for detection. To detect the symbols in $\mathbf{D}^{(2)}$, the MRC operation takes the form $\mathbf{Z}^{(2)}\dfrac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|}$. The final received symbols in noise and jamming interference are given by

$$\mathbf{U}^{(2)} = \mathbf{Z}^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|} = \mathbf{B}^{(2)} + \mathbf{n}_R^{(2)} + \mathbf{j}_R^{(2)}, \tag{5.54}$$

where $\mathbf{B}^{(2)} = \mathbf{z}_D^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|}$, $\mathbf{n}_R^{(2)} = \mathbf{n}_D^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|}$, and $\mathbf{j}_R^{(2)} = \mathbf{j}_D^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|}$. Hence, the dimension of $\mathbf{U}^{(2)}$ is $1 \times m_2$, which matches the original symbols sent in channel two. Without any noise and interference, the recovered vector $\mathbf{U}^{(2)}$ is

$$\mathbf{B}^{(2)} = \mathbf{z}_D^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|} = \mathbf{D}^{(2)} \left(\mathbf{H}^{(2)}\right)^t \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|} = \mathbf{D}^{(2)} \| \mathbf{h}^{(2)} \| \mathbf{I}_{m_2 \times m_2} = \| \mathbf{h}^{(2)} \| \mathbf{D}^{(2)}, \tag{5.55}$$

where $\mathbf{D}^{(2)}$, which is scaled by $\| \mathbf{h}^{(2)} \|$, is available for detection. Continuing from equation (5.51), we see that the recovered vector is given by

$$\begin{aligned}
\mathbf{B}^{(2)} &= \mathbf{z}_D^{(2)} \frac{\left(\mathbf{H}^{(2)}\right)^*}{\| \mathbf{h}^{(2)} \|} \\
&= \frac{1}{\| \mathbf{h}^{(2)} \|} \underbrace{\begin{bmatrix} h_1 D_1^{(2)} + h_2 D_2^{(2)} & h_2^* D_1^{(2)} - h_1^* D_2^{(2)} & h_1 D_3^{(2)} + h_2 D_4^{(2)} & h_2^* D_3^{(2)} - h_1^* D_4^{(2)} \end{bmatrix}}_{1 \times 2L_t} \underbrace{\begin{bmatrix} h_1^* & h_2^* & 0 & 0 \\ h_2 & -h_1 & 0 & 0 \\ 0 & 0 & h_1^* & h_2^* \\ 0 & 0 & h_2 & -h_1 \end{bmatrix}}_{2L_t \times m_2} \\
&= \frac{1}{\| \mathbf{h}^{(2)} \|} \begin{bmatrix} h_1^* h_1 D_1^{(2)} + h_1^* h_2 D_2^{(2)} + h_2^* h_2 D_1^{(2)} - h_1^* h_2 D_2^{(2)} \\ h_2^* h_1 D_1^{(2)} + h_2^* h_2 D_2^{(2)} - h_2^* h_1 D_1^{(2)} + h_1^* h_1 D_2^{(2)} \\ h_1^* h_1 D_3^{(2)} + h_1^* h_2 D_4^{(2)} + h_2^* h_2 D_3^{(2)} - h_1^* h_2 D_4^{(2)} \\ h_2^* h_1 D_3^{(2)} + h_2^* h_2 D_4^{(2)} - h_2^* h_1 D_3^{(2)} + h_1^* h_1 D_4^{(2)} \end{bmatrix}^t = \frac{1}{\| \mathbf{h}^{(2)} \|} \begin{bmatrix} D_1^{(2)} \left(h_1^* h_1 + h_2^* h_2\right) \\ D_2^{(2)} \left(h_2^* h_2 + h_1^* h_1\right) \\ D_3^{(2)} \left(h_1^* h_1 + h_2^* h_2\right) \\ D_4^{(2)} \left(h_2^* h_2 + h_1^* h_1\right) \end{bmatrix}^t \\
&= \frac{1}{\| \mathbf{h}^{(2)} \|} \begin{bmatrix} D_1^{(2)} \| \mathbf{h}^{(2)} \|^2 \\ D_2^{(2)} \| \mathbf{h}^{(2)} \|^2 \\ D_3^{(2)} \| \mathbf{h}^{(2)} \|^2 \\ D_4^{(2)} \| \mathbf{h}^{(2)} \|^2 \end{bmatrix}^t = \| \mathbf{h}^{(2)} \| \begin{bmatrix} D_1^{(2)} & D_2^{(2)} & D_3^{(2)} & D_4^{(2)} \end{bmatrix}.
\end{aligned} \tag{5.56}$$

E. NOISE AND INTERFERENCE POWER CALCULATIONS FOR MULTI-RATE CHANNELS

From the symbol recovery of channel one given in equation (4.13), (4.14) and (4.16), we know that the covariances of noise and jamming are the same as that of the fixed data rate in Chapter V, Section C. It is our interest to find the covariances of the noise and jamming for the higher data rate transmission of channel two, where the PN spreading gain for each symbol is affected by the concatenation during symbol recovery. In this case the spreading gain is $N/2^{l} = N/2$. Thus, we note that the spreading factor of a symbol of channel two is now $N/2$. Recall that after symbol recovery, the received measurement is (5.48). We need to find the covariances of $\mathbf{n}_D^{(2)} = 2\tilde{\boldsymbol{w}}^{(2)}\hat{\mathbf{n}}_S$ and $\mathbf{j}_D^{(2)} = 2\tilde{\boldsymbol{w}}^{(2)}\hat{\mathbf{j}}_S$. We also need to find the covariances of the MRC output $\mathbf{n}_R^{(2)} = \mathbf{n}_D^{(2)}\dfrac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|}$ and $\mathbf{j}_R^{(2)} = \mathbf{j}_D^{(2)}\dfrac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|}$.

1. Receiver Noise

If $\mathbf{n}_D^{(2)} = 2\tilde{\boldsymbol{w}}^{(2)}\hat{\mathbf{n}}_S$, then its covariance is given by

$$\operatorname{cov}\left(\mathbf{n}_D^{(2)}\right) = \mathrm{E}\left[\left(\mathbf{n}_D^{(2)}\right)^{\dagger}\mathbf{n}_D^{(2)}\right]. \tag{5.57}$$

Since $\hat{\mathbf{n}}_S$ is the mapped $\left(N/2\right)\times 2L_t$ matrix version of the noise matrix of $\mathbf{n}_S$, it is given by

$$\hat{\mathbf{n}}_S = \begin{bmatrix} c_1^* n_{11} & c_1^* n_{12} & \cdots & c_1^* n_{1L_t} & c^*_{\left(\frac{N}{2}+1\right)} n_{\left(\frac{N}{2}+1\right)1} & \cdots & c^*_{\left(\frac{N}{2}+1\right)} n_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^* n_{21} & c_2^* n_{22} & \cdots & c_2^* n_{2L_t} & c^*_{\left(\frac{N}{2}+2\right)} n_{\left(\frac{N}{2}+1\right)1} & \cdots & c^*_{\left(\frac{N}{2}+2\right)} n_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ c^*_{\frac{N}{2}} n_{\frac{N}{2}1} & c^*_{\frac{N}{2}} n_{\frac{N}{2}2} & \cdots & c^*_{\frac{N}{2}} n_{\frac{N}{2}L_t} & c_N^* n_{N1} & \cdots & c_N^* n_{NL_t} \end{bmatrix}. \tag{5.58}$$

For notational convenience, we define the repeated $1\times\left(N/2\right)$ Walsh vector in (4.17) as

$$\begin{aligned}\tilde{\boldsymbol{w}}^{(2)} &= \begin{bmatrix} w_1 & w_2 & \cdots & w_{\frac{N}{2}} \end{bmatrix} \\ &= \left\{ \tilde{\boldsymbol{w}}^{(2)} = \begin{bmatrix} \underbrace{\begin{bmatrix} w_1^{(2)} \cdots w_L^{(2)} \end{bmatrix}}_{1}, & \cdots, & \underbrace{\begin{bmatrix} w_1^{(1)} \cdots w_L^{(1)} \end{bmatrix}}_{b} \end{bmatrix}, \right.\end{aligned} \tag{5.59}$$

where $N/2 = bL = bM/2$. The de-spread noise vector is

$$\begin{aligned}\mathbf{n}_D^{(2)} = 2\tilde{\boldsymbol{w}}^{(2)}\hat{\mathbf{n}}_S &= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_{\frac{N}{2}} \end{bmatrix}}_{1\times\frac{N}{2}}\underbrace{\begin{bmatrix} c_1^* n_{11} & c_1^* n_{12} & \cdots & c_{\left(\frac{N}{2}+1\right)}^* n_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^* n_{21} & c_2^* n_{22} & \cdots & c_{\left(\frac{N}{2}+2\right)}^* n_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^* n_{\frac{N}{2}1} & c_{\frac{N}{2}}^* n_{\frac{N}{2}2} & \cdots & c_N^* n_{NL_t} \end{bmatrix}}_{\frac{N}{2}\times 2L_t} \\ &= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a c_a^* n_{a1} \\ \sum_{a=1}^{\frac{N}{2}} w_a c_a^* n_{a2} \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a c_{\left(\frac{N}{2}+a\right)}^* n_{\left(\frac{N}{2}+a\right)L_t} \end{bmatrix}^t}_{1\times 2L_t},\end{aligned} \tag{5.60}$$

where $\mathbf{n}_D^{(2)}$ is a $1\times 2L_t$ vector. Its Hermitian, which is a $2L_t\times 1$ vector, is given by

$$\left(\mathbf{n}_D^{(2)}\right)^\dagger = \left(2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{n}}_S\right)^\dagger = 2\widehat{\mathbf{n}}_S^\dagger\left(\widetilde{\boldsymbol{w}}^{(2)}\right)^\dagger$$

$$= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} n_{11}^* c_1 & n_{21}^* c_2 & \cdots & n_{\frac{N}{2}1}^* c_{\frac{N}{2}} \\ n_{12}^* c_1 & n_{22}^* c_2 & \cdots & n_{\frac{N}{2}2}^* c_{\frac{N}{2}} \\ \vdots & \vdots & \ddots & \vdots \\ n_{\left(\frac{N}{2}+1\right)L_t}^* c_{\left(\frac{N}{2}+1\right)} & n_{\left(\frac{N}{2}+2\right)L_t}^* c_{\left(\frac{N}{2}+2\right)} & \cdots & n_{NL_t}^* c_N \end{bmatrix}}_{2L_t\times\frac{N}{2}}\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{\frac{N}{2}} \end{bmatrix}}_{\frac{N}{2}\times 1} \tag{5.61}$$

$$= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} n_{a1}^* c_a w_a \\ \sum_{a=1}^{\frac{N}{2}} n_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} n_{\left(\frac{N}{2}+a\right)L_t}^* c_{\left(\frac{N}{2}+a\right)} w_a \end{bmatrix}}_{2L_t\times 1}.$$

The covariance is

$$\operatorname{cov}\left(\mathbf{n}_D^{(2)}\right) = \mathrm{E}\left[\left(\mathbf{n}_D^{(2)}\right)^\dagger \mathbf{n}_D^{(2)}\right]$$

$$= \mathrm{E}\left[\underbrace{\frac{2}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} n_{a1}^* c_a w_a \\ \sum_{a=1}^{\frac{N}{2}} n_{a2}^* c_a w_a \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} n_{\left(\frac{N}{2}+a\right)L_t}^* c_{\left(\frac{N}{2}+a\right)} w_a \end{bmatrix}}_{2L_t\times 1}\underbrace{\frac{2}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a c_a^* n_{a1} \\ \sum_{a=1}^{\frac{N}{2}} w_a c_a^* n_{a2} \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a c_{\left(\frac{N}{2}+a\right)}^* n_{\left(\frac{N}{2}+a\right)L_t} \end{bmatrix}^t}_{1\times 2L_t}\right] \tag{5.62}$$

$$= \frac{4}{\|\boldsymbol{c}\|^2}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_b^* n_{b1} n_{a1}^* c_a w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^* n_{\left(\frac{N}{2}+b\right)L_t} n_{a1}^* c_a w_a\right] \\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_b^* n_{b1} n_{a2}^* c_a w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^* n_{\left(\frac{N}{2}+b\right)L_t} n_{a2}^* c_a w_a\right] \\ \vdots & \ddots & \vdots \\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_b^* n_{b1} n_{\left(\frac{N}{2}+a\right)L_t}^* c_{\left(\frac{N}{2}+a\right)} w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} \mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^* n_{\left(\frac{N}{2}+b\right)L_t} n_{\left(\frac{N}{2}+a\right)L_t}^* c_{\left(\frac{N}{2}+a\right)} w_a\right] \end{bmatrix}.$$

Since the sampled receiver noise is AWGN, we know that

$$\begin{aligned} \mathrm{E}\left[n_{ij} n_{kl}^{*} \right] &= \sigma^2, \quad i = k,\ j = l \\ &= 0, \qquad \textit{otherwise} \end{aligned} \tag{5.63}$$

and since that $w_1^2 = w_2^2 = \cdots w_{\frac{N}{2}}^2 = 1$, $\sum_{a=1}^{\frac{N}{2}} \sum_{b=1}^{\frac{N}{2}} w_b c_b^{*} c_a w_a = \sum_{a=1}^{\frac{N}{2}} | c_a |^2$ and $\sum_{a=1}^{\frac{N}{2}} \sum_{b=1}^{\frac{N}{2}} w_b c_{\left(\frac{N}{2}+b\right)}^{*} c_{\left(\frac{N}{2}+a\right)} w_a = \sum_{a=1}^{\frac{N}{2}} | c_{\left(\frac{N}{2}+a\right)} |^2$. Thus, the covariance (5.62) reduces to

$$\mathrm{cov}\left(\mathbf{n}_D^{(2)} \right) = \frac{4}{\| \boldsymbol{c} \|^2} \begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} | c_a |^2 \sigma^2 & 0 & \cdots & 0 \\ 0 & \sum_{a=1}^{\frac{N}{2}} | c_a |^2 \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{a=1}^{\frac{N}{2}} | c_{\left(\frac{N}{2}+a\right)} |^2 \sigma^2 \end{bmatrix}, \tag{5.64}$$

and since, $\sum_{a=1}^{\frac{N}{2}} | c_a |^2 = \sum_{a=1}^{\frac{N}{2}} | c_{\left(\frac{N}{2}+a\right)} |^2 = \frac{\| \boldsymbol{c} \|^2}{2}$, the covariance is simply

$$\mathrm{cov}\left(\mathbf{n}_D^{(2)} \right) = 2\sigma_n^2 \mathbf{I}_{2L_t \times 2L_t}. \tag{5.65}$$

The MRC noise vector is given by $\mathbf{n}_R^{(2)} = \mathbf{n}_D^{(2)} \frac{\left(\mathbf{H}^{(2)} \right)^{*}}{\| \mathbf{h}^{(2)} \|}$, where $\mathbf{H}^{(2)}$ is a $2L_t \times m_2$ matrix. Hence, the covariance of the MRC noise is given by

$$\begin{aligned} \mathrm{cov}\left(\mathbf{n}_R^{(2)} \right) = \mathrm{E}\left[\left(\mathbf{n}_R^{(2)} \right)^{\dagger} \mathbf{n}_R^{(2)} \right] &= \mathrm{E}\left[\left(\mathbf{n}_D^{(2)} \frac{\left(\mathbf{H}^{(2)} \right)^{*}}{\| \mathbf{h}^{(2)} \|} \right)^{\dagger} \left(\mathbf{n}_D^{(2)} \frac{\left(\mathbf{H}^{(2)} \right)^{*}}{\| \mathbf{h}^{(2)} \|} \right) \right] \\ &= 2\sigma_n^2 \frac{\left(\mathbf{H}^{(2)} \right)^{t}}{\| \mathbf{h}^{(2)} \|} \mathbf{I}_{2L_t \times 2L_t} \frac{\left(\mathbf{H}^{(2)} \right)^{*}}{\| \mathbf{h}^{(2)} \|} \\ &= 2\sigma_n^2 \frac{\left(\mathbf{H}^{(2)} \right)^{t}}{\| \mathbf{h}^{(2)} \|} \frac{\left(\mathbf{H}^{(2)} \right)^{*}}{\| \mathbf{h}^{(2)} \|} = 2\sigma_n^2 \frac{\left(\mathbf{H}^{(2)} \right)^{\dagger} \mathbf{H}^{(2)}}{\| \mathbf{h}^{(2)} \|^2} \\ &= 2\sigma_n^2 \frac{\| \mathbf{h}^{(2)} \|^2}{\| \mathbf{h}^{(2)} \|^2} \mathbf{I}_{m_2 \times m_2} = 2\sigma_n^2 \mathbf{I}_{m_2 \times m_2}. \end{aligned} \tag{5.66}$$

2. Barrage Jamming

The complex interference sample for barrage noise jamming is distributed according to $\mathcal{CN}\left(0,\sigma_{B'}^2=\frac{\sigma_B^2}{N}\right)$. After the signal is received, the de-spread jamming $\left(N/2\right)\times 2L_t$ matrix $\widehat{\mathbf{j}}_{\mathrm{B}}$ for channel two is given as

$$\widehat{\mathbf{j}}_{\mathrm{B}}=\begin{bmatrix} c_1^*\varphi_{11} & c_1^*\varphi_{12} & \cdots & c_{\left(\frac{N}{2}+1\right)}^*\varphi_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^*\varphi_{21} & c_2^*\varphi_{22} & \cdots & c_{\left(\frac{N}{2}+2\right)}^*\varphi_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^*\varphi_{\frac{N}{2}1} & c_{\frac{N}{2}}^*\varphi_{\frac{N}{2}2} & \cdots & c_N^*\varphi_{NL_t} \end{bmatrix}. \tag{5.67}$$

Let $\mathbf{j}_B^{(2)}=2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{B}}$, then its covariance is given by

$$\mathrm{cov}\left(\mathbf{j}_B^{(2)}\right)=\mathrm{E}\left[\left(\mathbf{j}_B^{(2)}\right)^{\dagger}\mathbf{j}_B^{(2)}\right]. \tag{5.68}$$

The de-spread jamming vector is

$$\begin{aligned}\mathbf{j}_B^{(2)}=2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{B}}&=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_{\frac{N}{2}} \end{bmatrix}}_{1\times\frac{N}{2}}\underbrace{\begin{bmatrix} c_1^*\varphi_{11} & c_1^*\varphi_{12} & \cdots & c_{\left(\frac{N}{2}+1\right)}^*\varphi_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^*\varphi_{21} & c_2^*\varphi_{22} & \cdots & c_{\left(\frac{N}{2}+2\right)}^*\varphi_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^*\varphi_{\frac{N}{2}1} & c_{\frac{N}{2}}^*\varphi_{\frac{N}{2}2} & \cdots & c_N^*\varphi_{NL_t} \end{bmatrix}}_{\frac{N}{2}\times 2L_t} \\ &=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a c_a^*\varphi_{a1} \\ \sum_{a=1}^{\frac{N}{2}} w_a c_a^*\varphi_{a2} \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a c_{\left(\frac{N}{2}+a\right)}^*\varphi_{\left(\frac{N}{2}+a\right)L_t} \end{bmatrix}^t}_{1\times 2L_t},\end{aligned} \tag{5.69}$$

where $\mathbf{j}_B^{(2)}$ is a $1\times 2L_t$ vector. Its Hermitian, which is a $2L_t\times 1$ vector, is given by

$$\begin{aligned}\left(\mathbf{j}_B^{(2)}\right)^{\dagger} &= \left(2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{B}}\right)^{\dagger} = 2\widehat{\mathbf{j}}_{\mathrm{B}}^{\dagger}\left(\widetilde{\boldsymbol{w}}^{(2)}\right)^{\dagger} \\ &= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \varphi_{11}^{*}c_1 & \varphi_{21}^{*}c_2 & \cdots & \varphi_{\frac{N}{2}1}^{*}c_{\frac{N}{2}} \\ \varphi_{12}^{*}c_1 & \varphi_{22}^{*}c_2 & \cdots & \varphi_{\frac{N}{2}2}^{*}c_{\frac{N}{2}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{\left(\frac{N}{2}+1\right)L_t}^{*}c_{\left(\frac{N}{2}+1\right)} & \varphi_{\left(\frac{N}{2}+2\right)L_t}^{*}c_{\left(\frac{N}{2}+2\right)} & \cdots & \varphi_{NL_t}^{*}c_N \end{bmatrix}}_{2L_t\times\frac{N}{2}}\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{\frac{N}{2}} \end{bmatrix}}_{\frac{N}{2}\times 1} \\ &= \frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}}\varphi_{a1}^{*}c_a w_a \\ \sum_{a=1}^{\frac{N}{2}}\varphi_{a2}^{*}c_a w_a \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}}\varphi_{\left(\frac{N}{2}+a\right)L_t}^{*}c_{\left(\frac{N}{2}+a\right)} w_a \end{bmatrix}}_{2L_t\times 1}.\end{aligned} \tag{5.70}$$

The covariance is

$$\begin{aligned}\mathrm{cov}\left(\mathbf{j}_B^{(2)}\right) &= \mathrm{E}\left[\left(\mathbf{j}_B^{(2)}\right)^{\dagger}\mathbf{j}_B^{(2)}\right] \\ &= \mathrm{E}\left[\underbrace{\frac{2}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}}\varphi_{a1}^{*}c_a w_a \\ \sum_{a=1}^{\frac{N}{2}}\varphi_{a2}^{*}c_a w_a \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}}\varphi_{\left(\frac{N}{2}+a\right)L_t}^{*}c_{\left(\frac{N}{2}+a\right)} w_a \end{bmatrix}}_{2L_t\times 1}\underbrace{\frac{2}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}}w_a c_a^{*}\varphi_{a1} \\ \sum_{a=1}^{\frac{N}{2}}w_a c_a^{*}\varphi_{a2} \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}}w_a c_{\left(\frac{N}{2}+a\right)}^{*}\varphi_{\left(\frac{N}{2}+a\right)L_t} \end{bmatrix}^{t}}_{1\times 2L_t}\right] \\ &= \frac{4}{\|\boldsymbol{c}\|^2}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_b^{*}\varphi_{b1}\varphi_{a1}^{*}c_a w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^{*}\varphi_{\left(\frac{N}{2}+b\right)L_t}\varphi_{a1}^{*}c_a w_a\right] \\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_b^{*}\varphi_{b1}\varphi_{a2}^{*}c_a w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^{*}\varphi_{\left(\frac{N}{2}+b\right)L_t}\varphi_{a2}^{*}c_a w_a\right] \\ \vdots & \ddots & \vdots \\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_b^{*}\varphi_{b1}\varphi_{\left(\frac{N}{2}+a\right)L_t}^{*}c_{\left(\frac{N}{2}+a\right)} w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_b c_{\left(\frac{N}{2}+b\right)}^{*}\varphi_{\left(\frac{N}{2}+b\right)L_t}\varphi_{\left(\frac{N}{2}+a\right)L_t}^{*}c_{\left(\frac{N}{2}+a\right)} w_a\right] \end{bmatrix}.\end{aligned} \tag{5.71}$$

Since the complex noise sample is AWGN, we know that

$$\mathrm{E}\left[\varphi_{ij}\varphi_{kl}^{*}\right]=\frac{\sigma_B^2}{N}\quad i=k,\ j=l$$
$$=0,\qquad otherwise \tag{5.72}$$

and since that $w_1^2=w_2^2=\cdots w_{\frac{N}{2}}^2=1$, $\sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}w_b c_b^{*}c_a w_a=\sum_{a=1}^{\frac{N}{2}}|c_a|^2$ and $\sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}w_b c_{\left(\frac{N}{2}+b\right)}^{*}c_{\left(\frac{N}{2}+a\right)}w_a=\sum_{a=1}^{\frac{N}{2}}|c_{\left(\frac{N}{2}+a\right)}|^2$. Thus, the covariance (5.71) reduces to

$$\mathrm{cov}\left(\mathbf{j}_B^{(2)}\right)=\frac{4}{\|\boldsymbol{c}\|^2}\begin{bmatrix}\sum_{a=1}^{\frac{N}{2}}|c_a|^2\frac{\sigma_B^2}{N} & 0 & \cdots & 0\\ 0 & \sum_{a=1}^{\frac{N}{2}}|c_a|^2\frac{\sigma_B^2}{N} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sum_{a=1}^{\frac{N}{2}}|c_{\left(\frac{N}{2}+a\right)}|^2\frac{\sigma_B^2}{N}\end{bmatrix}, \tag{5.73}$$

and since $\sum_{a=1}^{\frac{N}{2}}|c_a|^2=\sum_{a=1}^{\frac{N}{2}}|c_{\left(\frac{N}{2}+a\right)}|^2=\frac{\|\boldsymbol{c}\|^2}{2}$, the covariance is simply

$$\mathrm{cov}\left(\mathbf{j}_B^{(2)}\right)=\frac{2\sigma_B^2}{N}\mathbf{I}_{2L_t\times 2L_t}. \tag{5.74}$$

The MRC noise vector is given by $\mathbf{j}_R^{(2)}=\mathbf{j}_B^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^{*}}{\|\mathbf{h}^{(2)}\|}$. Hence, the covariance of the MRC jamming is given by

$$\begin{aligned}\mathrm{cov}\left(\mathbf{j}_R^{(2)}\right)=\mathrm{E}\left[\left(\mathbf{j}_R^{(2)}\right)^{\dagger}\mathbf{j}_R^{(2)}\right]&=\mathrm{E}\left[\left(\mathbf{j}_B^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^{*}}{\|\mathbf{h}^{(2)}\|}\right)^{\dagger}\left(\mathbf{j}_B^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^{*}}{\|\mathbf{h}^{(2)}\|}\right)\right]\\ &=\frac{2\sigma_B^2}{N}\frac{\left(\mathbf{H}^{(2)}\right)^{t}}{\|\mathbf{h}^{(2)}\|}\mathbf{I}_{2L_t\times 2L_t}\frac{\left(\mathbf{H}^{(2)}\right)^{*}}{\|\mathbf{h}^{(2)}\|}\\ &=\frac{2\sigma_B^2}{N}\frac{\left(\mathbf{H}^{(2)}\right)^{t}}{\|\mathbf{h}^{(2)}\|}\frac{\left(\mathbf{H}^{(2)}\right)^{*}}{\|\mathbf{h}^{(2)}\|}=\frac{2\sigma_B^2}{N}\frac{\left(\mathbf{H}^{(2)}\right)^{\dagger}\mathbf{H}^{(2)}}{\|\mathbf{h}^{(2)}\|^2}\\ &=\frac{2\sigma_B^2}{N}\frac{\|\mathbf{h}^{(2)}\|^2}{\|\mathbf{h}^{(2)}\|^2}\mathbf{I}_{m_2\times m_2}=\frac{2\sigma_B^2}{N}\mathbf{I}_{m_2\times m_2}.\end{aligned} \tag{5.75}$$

3. Pulsed-Noise Jamming

For pulsed-noise jamming in channel two, the complex interference sample is distributed according to $\mathcal{CN}\left(0,\frac{\sigma_P^2}{\rho N}\right)$. After the signal is received, the pulsed-noise jamming $\left(N/2\right)\times 2L_t$ matrix is given as

$$\widehat{\mathbf{j}}_{\mathrm{P}}=\begin{bmatrix} c_1^*\psi_{11} & c_1^*\psi_{12} & \cdots & c_{\left(\frac{N}{2}+1\right)}^*\psi_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^*\psi_{21} & c_2^*\psi_{22} & \cdots & c_{\left(\frac{N}{2}+2\right)}^*\psi_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^*\psi_{\frac{N}{2}1} & c_{\frac{N}{2}}^*\psi_{\frac{N}{2}2} & \cdots & c_N^*\psi_{NL_t} \end{bmatrix}. \tag{5.76}$$

If $\mathbf{j}_P^{(2)}=2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{P}}$, then its covariance is given by

$$\mathrm{cov}\left(\mathbf{j}_P^{(2)}\right)=\mathrm{E}\left[\left(\mathbf{j}_P^{(2)}\right)^{\dagger}\mathbf{j}_P^{(2)}\right]. \tag{5.77}$$

The de-spread jamming vector is

$$\begin{aligned}\mathbf{j}_P^{(2)}=2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{P}}&=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_{\frac{N}{2}} \end{bmatrix}}_{1\times\frac{N}{2}}\underbrace{\begin{bmatrix} c_1^*\psi_{11} & c_1^*\psi_{12} & \cdots & c_{\left(\frac{N}{2}+1\right)}^*\psi_{\left(\frac{N}{2}+1\right)L_t} \\ c_2^*\psi_{21} & c_2^*\psi_{22} & \cdots & c_{\left(\frac{N}{2}+2\right)}^*\psi_{\left(\frac{N}{2}+2\right)L_t} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^*\psi_{\frac{N}{2}1} & c_{\frac{N}{2}}^*\psi_{\frac{N}{2}2} & \cdots & c_N^*\psi_{NL_t} \end{bmatrix}}_{\frac{N}{2}\times 2L_t} \\ &=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a c_a^*\psi_{a1} \\ \sum_{a=1}^{\frac{N}{2}} w_a c_a^*\psi_{a2} \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a c_{\left(\frac{N}{2}+a\right)}^*\psi_{\left(\frac{N}{2}+a\right)L_t} \end{bmatrix}^t}_{1\times 2L_t},\end{aligned} \tag{5.78}$$

where $\mathbf{j}_P^{(2)}$ is an $1\times 2L_t$ vector. Its Hermitian, which is a $2L_t\times 1$ vector, is given by

$$\left(\mathbf{j}_P^{(2)}\right)^\dagger=\left(2\widetilde{\boldsymbol{w}}^{(2)}\widehat{\mathbf{j}}_{\mathrm{P}}\right)^\dagger=2\widehat{\mathbf{j}}_{\mathrm{P}}^\dagger\left(\widetilde{\boldsymbol{w}}^{(2)}\right)^\dagger$$
$$=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\psi_{11}^*c_1 & \psi_{21}^*c_2 & \cdots & \psi_{\frac{N}{2}1}^*c_{\frac{N}{2}}\\ \psi_{12}^*c_1 & \psi_{22}^*c_2 & \cdots & \psi_{\frac{N}{2}2}^*c_{\frac{N}{2}}\\ \vdots & \vdots & \ddots & \vdots\\ \psi_{\left(\frac{N}{2}+1\right)L_t}^*c_{\left(\frac{N}{2}+1\right)} & \psi_{\left(\frac{N}{2}+2\right)L_t}^*c_{\left(\frac{N}{2}+2\right)} & \cdots & \psi_{NL_t}^*c_N\end{bmatrix}}_{2L_t\times\frac{N}{2}}\underbrace{\begin{bmatrix}w_1\\ w_2\\ \vdots\\ w_{\frac{N}{2}}\end{bmatrix}}_{\frac{N}{2}\times 1}$$
$$=\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\sum_{a=1}^{\frac{N}{2}}\psi_{a1}^*c_aw_a\\ \sum_{a=1}^{\frac{N}{2}}\psi_{a2}^*c_aw_a\\ \vdots\\ \sum_{a=1}^{\frac{N}{2}}\psi_{\left(\frac{N}{2}+a\right)L_t}^*c_{\left(\frac{N}{2}+a\right)}w_a\end{bmatrix}}_{2L_t\times 1}. \tag{5.79}$$

The covariance is

$$\mathrm{cov}\left(\mathbf{j}_P^{(2)}\right)=\mathrm{E}\left[\left(\mathbf{j}_P^{(2)}\right)^\dagger\mathbf{j}_P^{(2)}\right] \tag{5.80}$$
$$=\mathrm{E}\left[\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\sum_{a=1}^{\frac{N}{2}}\psi_{a1}^*c_aw_a\\ \sum_{a=1}^{\frac{N}{2}}\psi_{a2}^*c_aw_a\\ \vdots\\ \sum_{a=1}^{\frac{N}{2}}\psi_{\left(\frac{N}{2}+a\right)L_t}^*c_{\left(\frac{N}{2}+a\right)}w_a\end{bmatrix}}_{2L_t\times 1}\frac{2}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix}\sum_{a=1}^{\frac{N}{2}}w_ac_a^*\psi_{a1}\\ \sum_{a=1}^{\frac{N}{2}}w_ac_a^*\psi_{a2}\\ \vdots\\ \sum_{a=1}^{\frac{N}{2}}w_ac_{\left(\frac{N}{2}+a\right)}^*\psi_{\left(\frac{N}{2}+a\right)L_t}\end{bmatrix}^t}_{1\times 2L_t}\right]$$
$$=\frac{4}{\|\boldsymbol{c}\|^2}\begin{bmatrix}\sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_b^*\psi_{b1}\psi_{a1}^*c_aw_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_{\left(\frac{N}{2}+b\right)}^*\psi_{\left(\frac{N}{2}+b\right)L_t}\psi_{a1}^*c_aw_a\right]\\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_b^*\psi_{b1}\psi_{a2}^*c_aw_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_{\left(\frac{N}{2}+b\right)}^*\psi_{\left(\frac{N}{2}+b\right)L_t}\psi_{a2}^*c_aw_a\right]\\ \vdots & \ddots & \vdots\\ \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_b^*\psi_{b1}\psi_{\left(\frac{N}{2}+a\right)L_t}^*c_{\left(\frac{N}{2}+a\right)}w_a\right] & \cdots & \sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}}\mathrm{E}\left[w_bc_{\left(\frac{N}{2}+b\right)}^*\psi_{\left(\frac{N}{2}+b\right)L_t}\psi_{\left(\frac{N}{2}+a\right)L_t}^*c_{\left(\frac{N}{2}+a\right)}w_a\right]\end{bmatrix}.$$

Since the complex noise sample is AWGN, we know that

$$\begin{aligned} \mathrm{E}\left[\psi_{ij}\psi_{kl}^{*}\right] &= \frac{\sigma_P^2}{\rho N} \quad i=k,\ j=l \\ &= 0, \qquad \textit{otherwise} \end{aligned} \tag{5.81}$$

and since that $w_1^2 = w_2^2 = \cdots w_{\frac{N}{2}}^2 = 1$, $\sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} w_b c_b^* c_a w_a = \sum_{a=1}^{\frac{N}{2}} |\, c_a\,|^2$ and $\sum_{a=1}^{\frac{N}{2}}\sum_{b=1}^{\frac{N}{2}} w_b c^*_{\left(\frac{N}{2}+b\right)} c_{\left(\frac{N}{2}+a\right)} w_a = \sum_{a=1}^{\frac{N}{2}} |\, c_{\left(\frac{N}{2}+a\right)}\,|^2$. Thus, the covariance (5.80) reduces to

$$\mathrm{cov}\left(\mathbf{j}_P^{(2)}\right) = \frac{4}{\|\boldsymbol{c}\|^2}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} |\, c_a\,|^2 \frac{\sigma_P^2}{\rho N} & 0 & \cdots & 0 \\ 0 & \sum_{a=1}^{\frac{N}{2}} |\, c_a\,|^2 \frac{\sigma_P^2}{\rho N} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{a=1}^{\frac{N}{2}} |\, c_{\left(\frac{N}{2}+a\right)}\,|^2 \frac{\sigma_P^2}{\rho N} \end{bmatrix}, \tag{5.82}$$

and since $\sum_{a=1}^{\frac{N}{2}} |\, c_a\,|^2 = \sum_{a=1}^{\frac{N}{2}} |\, c_{\left(\frac{N}{2}+a\right)}\,|^2 = \frac{\|\boldsymbol{c}\|^2}{2}$, the covariance is simply

$$\mathrm{cov}\left(\mathbf{j}_P^{(2)}\right) = \frac{2\sigma_P^2}{\rho N}\mathbf{I}_{2L_t\times 2L_t}. \tag{5.83}$$

The MRC noise vector is given by $\mathbf{j}_S^{(2)} = \mathbf{j}_P^{(2)} \dfrac{\left(\mathbf{H}^{(2)}\right)^*}{\|\,\mathbf{h}^{(2)}\,\|}$. Hence, the covariance of the MRC jamming is given by

$$\begin{aligned} \mathrm{cov}\left(\mathbf{j}_S^{(2)}\right) = \mathrm{E}\left[\left(\mathbf{j}_S^{(2)}\right)^{\dagger}\mathbf{j}_S^{(2)}\right] &= \mathrm{E}\left[\left(\mathbf{j}_P^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\,\mathbf{h}^{(2)}\,\|}\right)^{\dagger}\left(\mathbf{j}_P^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\,\mathbf{h}^{(2)}\,\|}\right)\right] \\ &= \frac{2\sigma_P^2}{\rho N}\frac{\left(\mathbf{H}^{(2)}\right)^t}{\|\,\mathbf{h}^{(2)}\,\|}\mathbf{I}_{2L_t\times 2L_t}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\,\mathbf{h}^{(2)}\,\|} \\ &= \frac{2\sigma_P^2}{\rho N}\frac{\left(\mathbf{H}^{(2)}\right)^t}{\|\,\mathbf{h}^{(2)}\,\|}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\,\mathbf{h}^{(2)}\,\|} = \frac{2\sigma_P^2}{\rho N}\frac{\left(\mathbf{H}^{(2)}\right)^{\dagger}\mathbf{H}^{(2)}}{\|\,\mathbf{h}^{(2)}\,\|^2} \\ &= \frac{2\sigma_P^2}{\rho N}\frac{\|\,\mathbf{h}^{(2)}\,\|^2}{\|\,\mathbf{h}^{(2)}\,\|^2}\mathbf{I}_{m_2\times m_2} = \frac{2\sigma_P^2}{\rho N}\mathbf{I}_{m_2\times m_2}. \end{aligned} \tag{5.84}$$

4. Tone Jamming

Recall from (5.41) that the interference prior to de-spreading is $\mathbf{J}=\begin{bmatrix} \mathbf{j}_1 & \mathbf{j}_2 & \cdots & \mathbf{j}_N \end{bmatrix}^t$, and an arbitrary jamming interference $\mathbf{j}$ is given by $\mathbf{j}=\begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_{L_t} \end{bmatrix}=\frac{A_J}{2}\begin{bmatrix} e^{-j\theta_1} & e^{-j\theta_2} & \cdots & e^{-j\theta_{L_t}} \end{bmatrix}$, where A_J is the amplitude of the carrier tone. After de-spreading, the tone jamming $\left(N/2\right)\times 2L_t$ matrix is given as

$$\widehat{\mathbf{j}}_{\mathrm{T}}=\frac{A_J}{2}\begin{bmatrix} c_1^* e^{-j\theta_{11}} & c_1^* e^{-j\theta_{21}} & \cdots & c_{\left(\frac{N}{2}+1\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+1\right)}} \\ c_2^* e^{-j\theta_{12}} & c_2^* e^{-j\theta_{22}} & \cdots & c_{\left(\frac{N}{2}+2\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+2\right)}} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^* e^{-j\theta_{1\frac{N}{2}}} & c_{\frac{N}{2}}^* e^{-j\theta_{2\frac{N}{2}}} & \cdots & c_N^* e^{-j\theta_{L_tN}} \end{bmatrix}, \tag{5.85}$$

and the de-spread jamming vector is given by

$$\begin{aligned} \mathbf{j}_Q^{(2)} &= 2\widetilde{\boldsymbol{w}}\widehat{\mathbf{j}}_{\mathrm{T}} \\ &= \frac{A_J}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} w_1 & w_2 & \cdots & w_{\frac{N}{2}} \end{bmatrix}}_{1\times\frac{N}{2}}\underbrace{\begin{bmatrix} c_1^* e^{-j\theta_{11}} & c_1^* e^{-j\theta_{21}} & \cdots & c_{\left(\frac{N}{2}+1\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+1\right)}} \\ c_2^* e^{-j\theta_{12}} & c_2^* e^{-j\theta_{22}} & \cdots & c_{\left(\frac{N}{2}+2\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+2\right)}} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\frac{N}{2}}^* e^{-j\theta_{1\frac{N}{2}}} & c_{\frac{N}{2}}^* e^{-j\theta_{2\frac{N}{2}}} & \cdots & c_N^* e^{-j\theta_{L_tN}} \end{bmatrix}}_{\frac{N}{2}\times 2L_t} \\ &= \frac{A_J}{\|\boldsymbol{c}\|}\begin{bmatrix} w_1 c_1^* e^{-j\theta_{11}} + w_2 c_2^* e^{-j\theta_{12}} + \cdots + w_{\frac{N}{2}} c_{\frac{N}{2}}^* e^{-j\theta_{1\frac{N}{2}}} \\ w_1 c_1^* e^{-j\theta_{21}} + w_2 c_2^* e^{-j\theta_{22}} + \cdots + w_{\frac{N}{2}} c_{\frac{N}{2}}^* e^{-j\theta_{2\frac{N}{2}}} \\ \vdots \\ w_1 c_{\left(\frac{N}{2}+1\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+1\right)}} + w_2 c_{\left(\frac{N}{2}+2\right)}^* e^{-j\theta_{L_t\left(\frac{N}{2}+2\right)}} + \cdots + w_{\frac{N}{2}} c_N^* e^{-j\theta_{L_tN}} \end{bmatrix}^t. \end{aligned} \tag{5.86}$$

It can be shown that the tone vector is given by

$$\mathbf{j}_Q = \frac{A_J}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a\left(c_{Ia}^* - jc_{Qa}^*\right)\left(\cos\theta_{1a} - j\sin\theta_{1a}\right) \\ \sum_{a=1}^{\frac{N}{2}} w_a\left(c_{Ia}^* - jc_{Qa}^*\right)\left(\cos\theta_{2a} - j\sin\theta_{2a}\right) \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a\left(c^*_{I\left(\frac{N}{2}+a\right)} - jc^*_{Q\left(\frac{N}{2}+a\right)}\right)\left(\cos\theta_{L_t\left(\frac{N}{2}+a\right)} - j\sin\theta_{L_t\left(\frac{N}{2}+a\right)}\right) \end{bmatrix}^t \tag{5.87}$$

$$= \frac{A_J}{\|\boldsymbol{c}\|}\begin{bmatrix} \sum_{a=1}^{\frac{N}{2}} w_a\left(c_{Ia}\cos\theta_{1a} - c_{Qa}\sin\theta_{1a} - jc_{Qa}\cos\theta_{1a} - jc_{Ia}\sin\theta_{1a}\right) \\ \sum_{a=1}^{\frac{N}{2}} w_a\left(c_{Ia}\cos\theta_{2a} - c_{Qa}\sin\theta_{2a} - jc_{Qa}\cos\theta_{2a} - jc_{Ia}\sin\theta_{2a}\right) \\ \vdots \\ \sum_{a=1}^{\frac{N}{2}} w_a\left(c_{I\left(\frac{N}{2}+a\right)}\cos\theta_{L_t\left(\frac{N}{2}+a\right)} - c_{Q\left(\frac{N}{2}+a\right)}\sin\theta_{L_t\left(\frac{N}{2}+a\right)} - jc_{Q\left(\frac{N}{2}+a\right)}\cos\theta_{L_t\left(\frac{N}{2}+a\right)} - jc_{I\left(\frac{N}{2}+a\right)}\sin\theta_{L_t\left(\frac{N}{2}+a\right)}\right) \end{bmatrix}^t.$$

Note that $c_{Ia}^* = c_{Ia}$, $c_{Qa}^* = c_{Qa}$, $c^*_{I\left(\frac{N}{2}+a\right)} = c_{I\left(\frac{N}{2}+a\right)}$ and $c^*_{Q\left(\frac{N}{2}+a\right)} = c_{Q\left(\frac{N}{2}+a\right)}$. Since the chips are real constants of 1 and -1, $\tilde{\boldsymbol{w}}$ can be all 1s, or even number of 1s and -1s. We assume that there is an even number of chips in the PN code since the Walsh sequences are always even. Furthermore, assume that the number of 1s outnumbers -1s by α such that $\sum_{a=1}^{\frac{N}{2}} w_a c_{Ia} = \sum_{a=1}^{\frac{N}{2}} w_a c_{Qa} = \sum_{a=1}^{\frac{N}{2}} w_a c_{I\left(\frac{N}{2}+1\right)} = \sum_{a=1}^{\frac{N}{2}} w_a c_{Q\left(\frac{N}{2}+1\right)} = \alpha$. Then (5.87) can be simplified to

$$\mathbf{j}_Q = \left(\frac{A_J}{\|\boldsymbol{c}\|}\right)(\alpha)\begin{bmatrix} \left(\cos\theta_1 - j\sin\theta_1\right) - \left(j\cos\theta_1 + \sin\theta_1\right) \\ \left(\cos\theta_2 - j\sin\theta_2\right) - \left(j\cos\theta_2 + \sin\theta_2\right) \\ \vdots \\ \left(\cos\theta_{L_t} - j\sin\theta_{L_t}\right) - \left(j\cos\theta_{L_t} + \sin\theta_{L_t}\right) \end{bmatrix}^t \tag{5.88}$$

$$= \frac{\alpha A_J}{\|\boldsymbol{c}\|}\begin{bmatrix} e^{-j\theta_1} - je^{-j\theta_1} \\ e^{-j\theta_2} - je^{-j\theta_2} \\ \vdots \\ e^{-j\theta_{L_t}} - je^{-j\theta_{L_t}} \end{bmatrix}^t.$$

Note that if $\alpha = 0$, the tone jamming is completely nulled out. Thus, the covariance is given by

$$
\begin{aligned}
\mathrm{cov}\left(\mathbf{j}_Q\right) &= \mathrm{E}\left[\left(\mathbf{j}_Q\right)^{\dagger}\mathbf{j}_Q\right] \\
&= \mathrm{E}\left[\frac{\alpha A_J}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} e^{j\theta_1}+je^{j\theta_1} \\ e^{j\theta_2}+je^{j\theta_2} \\ \vdots \\ e^{j\theta_{L_t}}+je^{j\theta_{L_t}} \end{bmatrix}}_{2L_t\times 1}\frac{\alpha A_J}{\|\boldsymbol{c}\|}\underbrace{\begin{bmatrix} e^{-j\theta_1}-je^{-j\theta_1} \\ e^{-j\theta_2}-je^{-j\theta_2} \\ \vdots \\ e^{-j\theta_{L_t}}-je^{-j\theta_{L_t}} \end{bmatrix}^t}_{1\times 2L_t}\right] \\
&= \left(\frac{\alpha A_J}{\|\boldsymbol{c}\|}\right)^2\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{2L_t\times 2L_t} \\
&= \frac{\alpha^2 A_J^2}{\|\boldsymbol{c}\|^2}\mathrm{I}_{2L_t\times 2L_t} = \frac{\alpha^2 A_J^2}{2N}\mathrm{I}_{2L_t\times 2L_t},
\end{aligned}
\tag{5.89}
$$

where $\mathrm{I}_{2L_t\times 2L_t}$ is an identity matrix, and $\|\boldsymbol{c}\|^2 = 2N$. The MRC interference vector is given by $\mathbf{j}_T = \mathbf{j}_Q\dfrac{\mathbf{H}^*}{\|\mathbf{h}\|}$. Hence, the covariance of the MRC noise is given by

$$
\begin{aligned}
\mathrm{cov}\left(\mathbf{j}_T^{(2)}\right) = \mathrm{E}\left[\left(\mathbf{j}_T^{(2)}\right)^{\dagger}\mathbf{j}_T^{(2)}\right] &= \mathrm{E}\left[\left(\mathbf{j}_Q^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|}\right)^{\dagger}\left(\mathbf{j}_Q^{(2)}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|}\right)\right] \\
&= \frac{\alpha^2 A_J^2}{2N}\frac{\left(\mathbf{H}^{(2)}\right)^t}{\|\mathbf{h}^{(2)}\|}\mathrm{I}_{2L_t\times 2L_t}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|} \\
&= \frac{\alpha^2 A_J^2}{2N}\frac{\left(\mathbf{H}^{(2)}\right)^t}{\|\mathbf{h}^{(2)}\|}\frac{\left(\mathbf{H}^{(2)}\right)^*}{\|\mathbf{h}^{(2)}\|} = \frac{\alpha^2 A_J^2}{2N}\frac{\left(\mathbf{H}^{(2)}\right)^{\dagger}\mathbf{H}^{(2)}}{\|\mathbf{h}^{(2)}\|^2} \\
&= \frac{\alpha^2 A_J^2}{2N}\frac{\|\mathbf{h}^{(2)}\|^2}{\|\mathbf{h}^{(2)}\|^2}\mathrm{I}_{m_2\times m_2} = \frac{\alpha^2 A_J^2}{2N}\mathrm{I}_{m_2\times m_2}.
\end{aligned}
\tag{5.90}
$$

F. SUMMARY

In this chapter, we discussed the procedure for complex spreading of MIMO signals with orthogonal covering for fixed and multi-rate data transmission. The OSTBC signals in Rayleigh fading, noise and interference as well as MRC operation were examined. The noise and jamming covariances calculations were covered in Section C and E for the fixed and multi-rate channels. For multi-rate channels, if the rate for channel two is 2^l times the rate of channel one, the variances are 2^l times that of channel one. This is a result of the reduced spreading factor N by 2^l for each symbol.

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VI. NUMERICAL ANALYSIS

We now use the general results derived from the previous sections in this chapter to derive the BER. The signal model is a 32QAM complex spreading MIMO system in Rayleigh fading with the three types of jamming. MRC is assumed in the receiver. The diversity gain for MIMO is $L = L_t L_r$. Hence, to maintain a proper comparison, the symbol energy is adjusted depending on the number of transmit antennas and the rate of space-time code. Here, we use $L_t = 2$ and Alamouti code with code rate R = 1; thus, the symbol energy is multiplied by $R/L_t = 1/2$.

A. SINR AND BER RESULTS FOR FIXED DATA RATE CHANNEL

The BER results are reported using the results in Chapter V for a fixed data rate channel.

1. BER for Rayleigh Fading with Diversity [1]

When the sub-channel tap magnitudes $|h_i|$ are Rayleigh distributed with normalized mean-square value $\mathrm{E}\left(|h_i|^2\right) = 1$, the combining channel tap magnitude has a $\mathcal{X}^2$-density function with 2L degrees of freedom. In an L-fold diversity system, the diversity order is $L = L_t L_r$, where L_t and L_t are the number of transmitter and receiver antennas. The bit error probability is given by [1]

$$P_b \approx \frac{N_n}{\log_2 M}\left(\frac{1-\mu}{2}\right)^L \sum_{l=0}^{L-1}\binom{L-1+l}{l}\left(\frac{1+\mu}{2}\right)^l, \tag{6.1}$$

where N_n and M depends on the type of modulation and μ is given by [1]

$$\mu = \sqrt{\frac{\alpha \mathbf{SNR}}{1+\alpha \mathbf{SNR}}}. \tag{6.2}$$

The diversity signal-to-noise ratio is **SNR** and is a modulation-dependent parameter based on the minimum Euclidean distance of the I-Q signal set. Thus, α**SNR** is given by [1]

$$\alpha\mathbf{SNR} = \frac{1}{2}\left(\frac{d_{\min}}{2\sigma}\right)^2, \tag{6.3}$$

where σ is the standard deviation of noise.

2. MQAM [1]

The minimum Euclidean distance of MQAM is $d_{\min} = \sqrt{6\varepsilon_s/(M-1)}$, where ε_s is the diversity energy given by $\varepsilon_s = E_s / L_t$. The average symbol energy is $E_s = E_b \log_2 M$, and E_b is the bit energy. The N_n for MQAM is given as $N_n = 4 - 4/\sqrt{M}$.

3. BER for Barrage Jamming

In 32QAM, there are M = 32 symbols, $N_n = 4 - 4/\sqrt{M} = 4 - 4/\sqrt{32}$ and $d_{\min} = \sqrt{6\varepsilon_s/(M-1)} = \sqrt{6\varepsilon_s/31}$. The noise variance is given in terms of spectral density as $\sigma_N^2 = N_0/2$ and the jamming variance is $\sigma_J^2 = J_0^{'}/2$. For received signal with noise and jamming, equation in (6.3) is modified to

$$\begin{aligned}\alpha\mathbf{SJNR} = \frac{1}{2}\left(\frac{d_{\min}}{2\sigma_T}\right)^2 = \frac{1}{2}\left(\frac{\sqrt{\frac{6\varepsilon_s}{31}}}{2\sigma_T}\right)^2 = \frac{1}{2}\frac{6\varepsilon_s}{31*4\frac{\left(N_0 + J_0^{'}\right)}{2}} \\ = \frac{15E_b}{31\frac{\left(N_0 + J_0^{'}\right)}{2}} = \frac{15}{62L_t\left[\left(\frac{E_b}{N_0}\right)^{-1} + \left(\frac{J_0^{'}}{N_0}\right)^{-1}\right]},\end{aligned} \tag{6.4}$$

where **SJNR** is signal-to-jamming-noise ratio. In the despread system, the sample jamming variance which corresponds to the result in (5.25) is expressed as $\sigma_B^2/N = J_0/2N$, where $J_0^{'} = J_0/N$. Thus, μ in (6.2) is given by

$$\mu = \sqrt{\frac{\alpha \mathbf{SNR}}{1+\alpha \mathbf{SNR}}} = \sqrt{\frac{1}{\frac{1}{\alpha \mathbf{SJNR}}+1}} = \sqrt{\frac{1}{\frac{62}{15} L_t \left[\left(\frac{E_b}{N_0} \right)^{-1} + \left(\frac{J_0'}{N_0} \right)^{-1} \frac{1}{N} \right] + 1}}. \qquad (6.5)$$

Hence, (6.5) is used in (6.1) to generate the BER results shown in Figures 20–22.

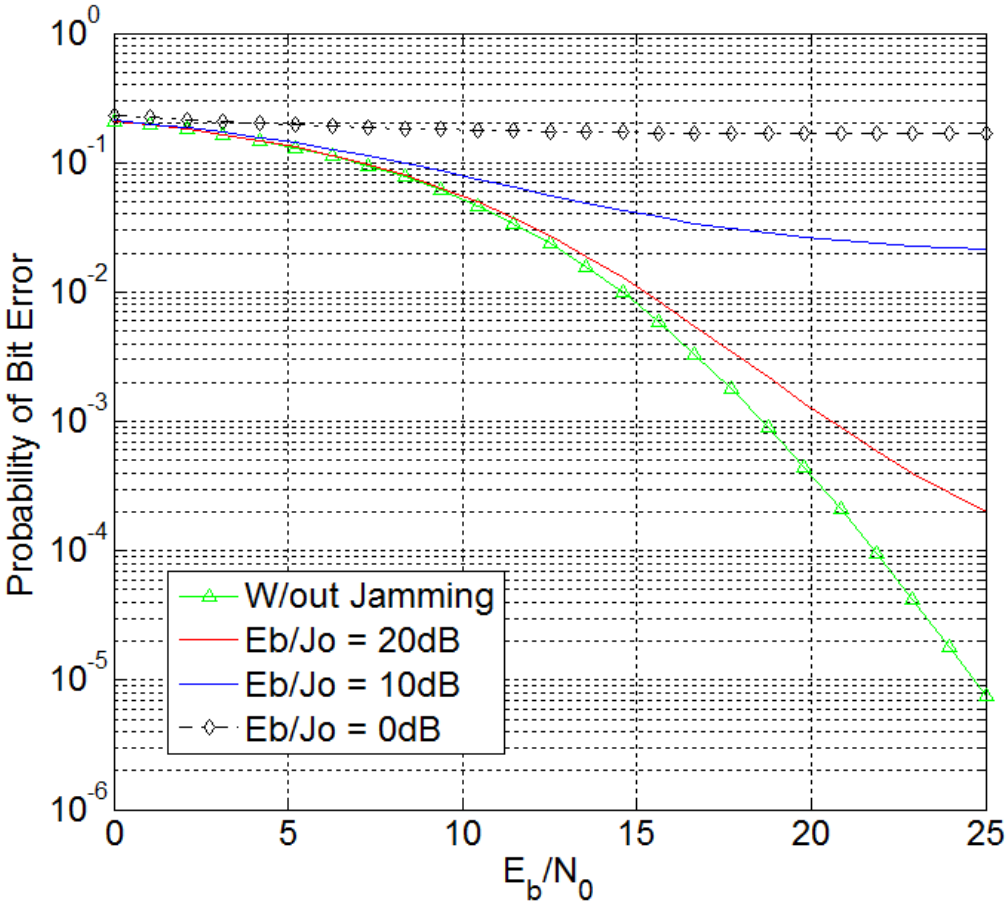

Figure 20. The BER for a MIMO (2×2) system with barrage jamming and without complex spreading are shown.

The BER for MIMO (2×2) system with jamming and no spreading is shown in Figure 20. From the plot, the BER decreases as the jamming power increases, i.e., decreasing E_b/J_0. The BER is nearly flats for $E_b/J_0 = 0$ dB.

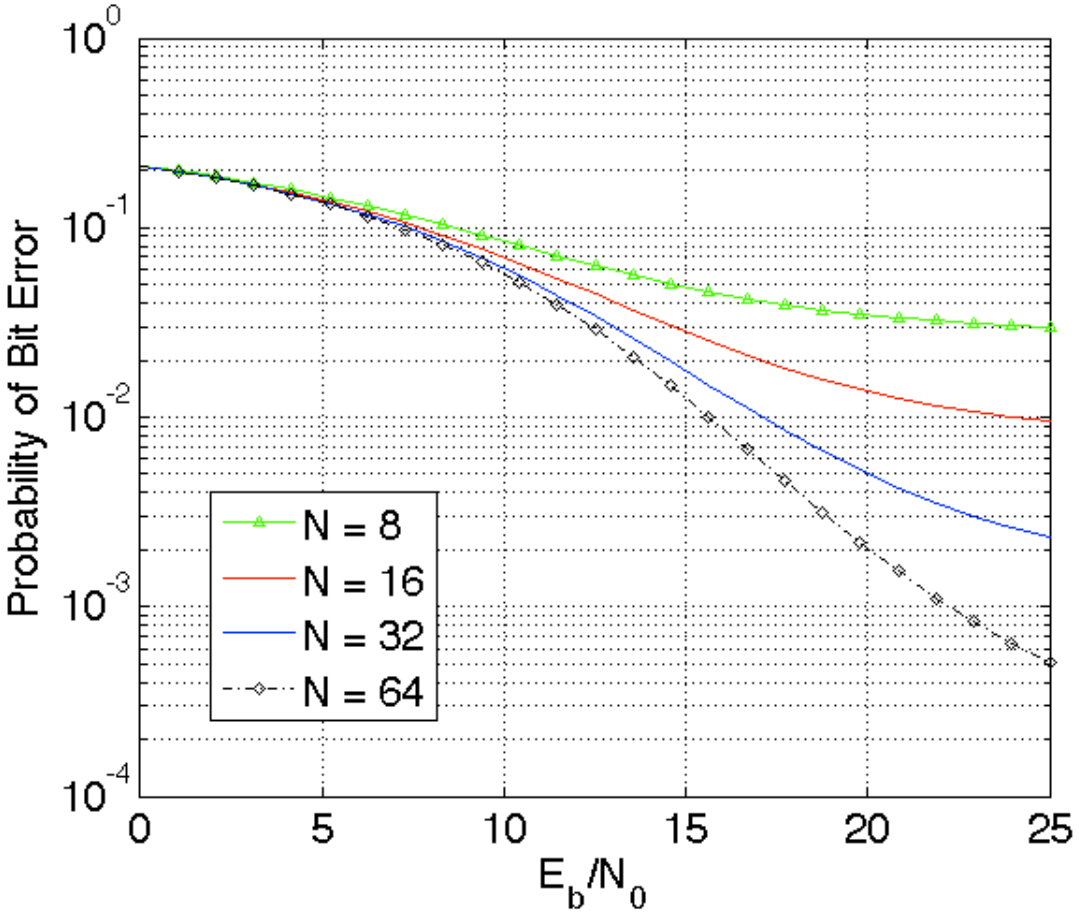

Figure 21. The BER for a MIMO (2×2) system with barrage jamming and complex spreading are shown.

In Figure 21, the BER for MIMO (2×2) system with $E_b/J_0 = 0$ dB are shown for various complex spreading factors *N*. The BER performance improves as *N* is increased, which illustrates the positive effect of spreading against jamming.

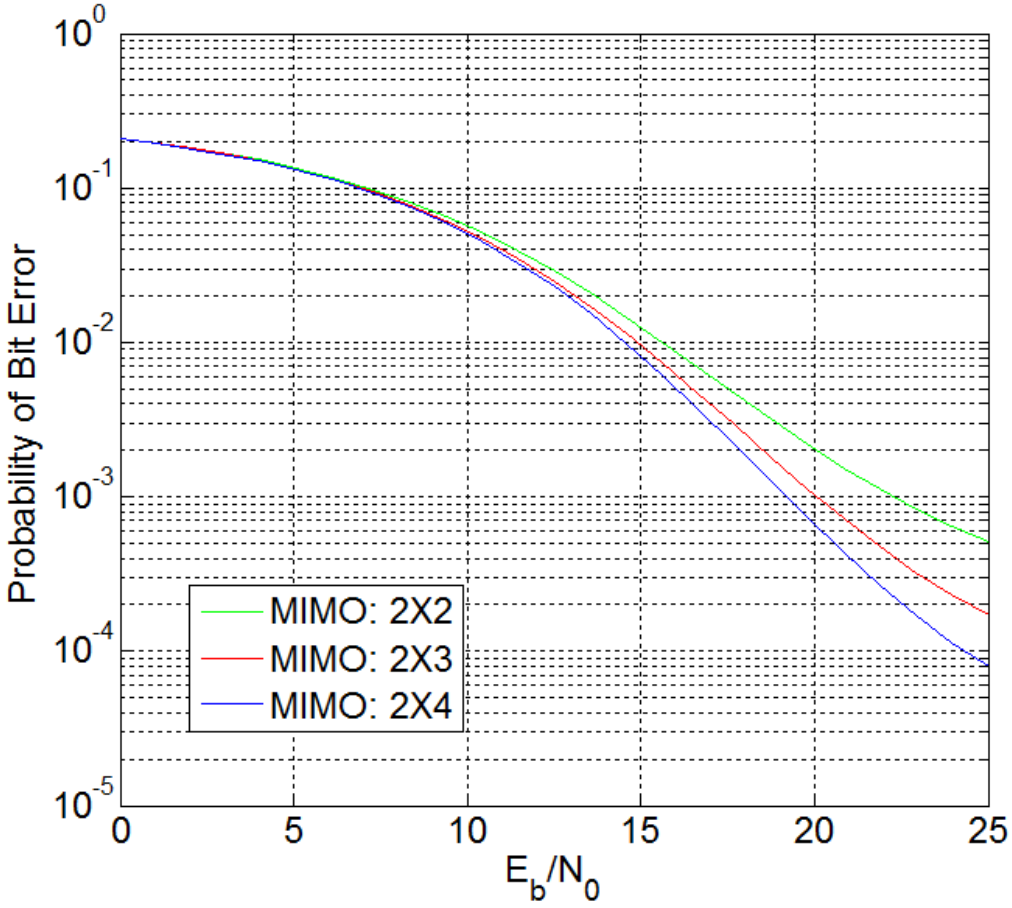

Figure 22. The BER are shown for the following MIMO systems: 2×2, 2×3 and 2×4 with barrage jamming.

From Figure 22, we can see the improvement of BER performance as the number of receiver antennas increase. Here we use $L_t = 2$ and $L_r = 2,3,4$, and note the BER improvement as L_r increases for $E_b/N_0 > 10$ dB.

4. BER for Pulsed-Noise Jamming

For pulsed-noise jamming, the sample jamming variance which corresponds to the result in (5.34) is expressed as $\sigma_P^2/\rho N = J_0/2\rho N$, where $J_0^{'} = J_0/\rho N$. Here we consider the case where the jamming power is much greater than the receiver noise, i.e.,

$\left(E_b/J_0^{'}\right)^{-1} \gg \left(E_b/N_0\right)^{-1}$. Thus, the BER is dominated by the jamming signal, and SJNR becomes signal-to-jamming ratio (SJR). Hence, the parameter μ simplifies to

$$\mu = \sqrt{\frac{1}{\frac{1}{\alpha \mathbf{SJR}} + 1}} = \sqrt{\frac{1}{\frac{62L_t}{15\rho N}\left(\frac{J_0^{'}}{N_0}\right)^{-1} + 1}}. \tag{6.6}$$

The BER results are shown in Figures 23–25.

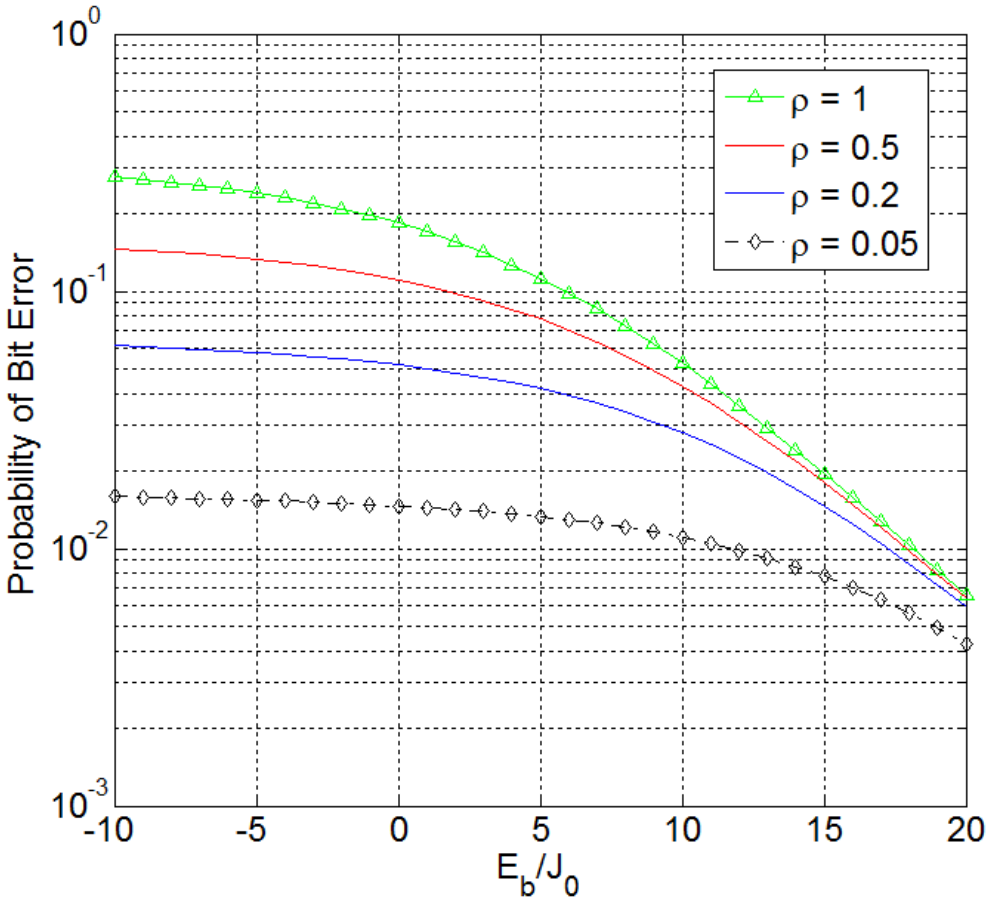

Figure 23. The BER for a SISO system with pulsed-noise jamming and without complex spreading are shown.

In Figure 23, the BER curves versus E_b / J_0 as function of increasing ρ for a single-input-single-output (SISO) system with pulsed jamming are shown. Again, it can

be seen that, without spreading, the probability of bit error is quite high in the presence of jamming. We can see that the BER worsens as ρ increases.

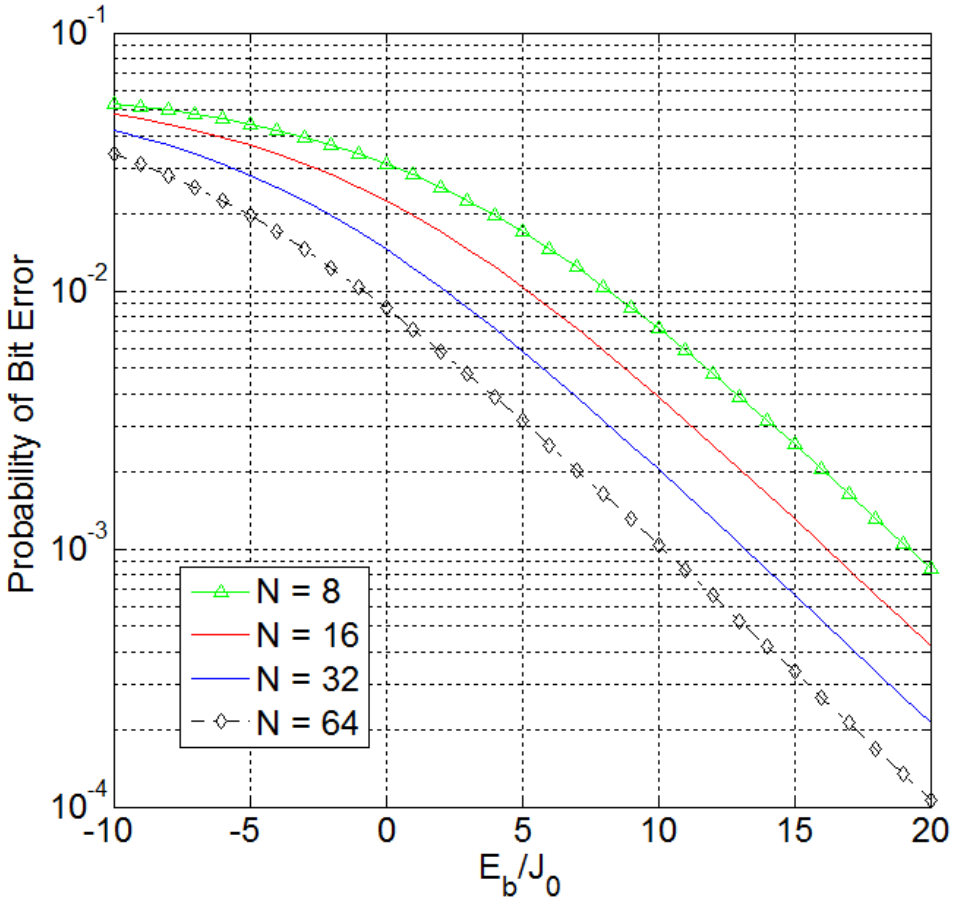

Figure 24. The BER for a SISO system with pulsed-noise jamming and complex spreading are shown.

In Figure 24, we fix $\rho = 0.2$ and vary the complex spreading for N = 8, 16, 32, 64. As expected, the BER improves as N increases.

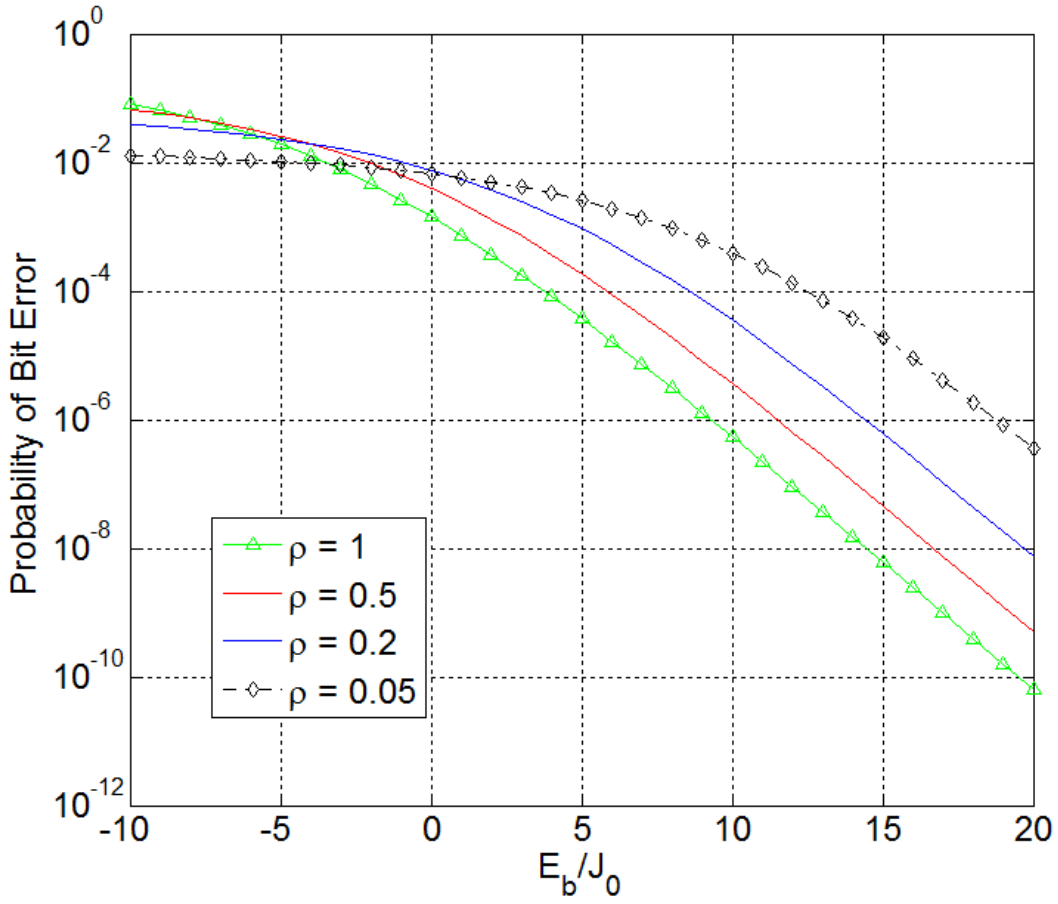

Figure 25. The BER for a MIMO (2×2) system with pulsed-noise jamming and increasing ρ are shown.

In Figure 25, we note the BERs improve with a 2×2 diversity system. However, the BER worsens as ρ decreases, which is opposite of what is observed in Figure 23. Thus, a jammer with small ρ is more effective against a MIMO system in a Rayleigh fading environment for high signal-to-jamming ratios.

5. BER for Tone Jamming

For tone jamming, the modulation-scaled SJNR is given by $\alpha\text{SJNR} = d_{\min}^2 \big/ \big(2\big) 4\sigma_T^2$ and the sample jamming variance which corresponds to the result in (5.40) is expressed as $J_0^{'} = \alpha^2 A_J^2 \big/ 4N$. For 32QAM,

$$\alpha\text{SJNR} = \frac{1}{2} \frac{6\varepsilon_s}{31*4\left(\frac{N_0}{2} + \frac{\alpha^2 A_J^2}{4N} \right)} = \frac{15E_b}{124L_t \left(\frac{N_0}{2} + \frac{\alpha^2 A_J^2}{4N} \right)}. \tag{6.7}$$

Recall that the bit energy is $E_b = A^2 N \big/ 2$, where A is the carrier amplitude; thus,

$$\alpha\text{SJNR} = \frac{15}{62L_t \left[\left(\frac{E_b}{N_0} \right)^{-1} + \left(\frac{A^2}{A_J^2} \right)^{-1} \frac{\alpha^2}{N^2} \right]}. \tag{6.8}$$

Hence, μ is given by

$$\mu = \sqrt{\frac{1}{\frac{1}{\alpha\textbf{SJNR}} + 1}} = \sqrt{\frac{1}{\frac{62L_t}{15} \left[\left(\frac{E_b}{N_0} \right)^{-1} + \left(\frac{A^2}{A_J^2} \right)^{-1} \frac{\alpha^2}{N^2} \right] + 1}}. \tag{6.9}$$

The BER results are shown in Figures 26–28. Note that if $\alpha = 0$, the tone jamming is completely nulled out. Thus, we take the minimum value, $\alpha = 2$, to analyze the jamming performance.

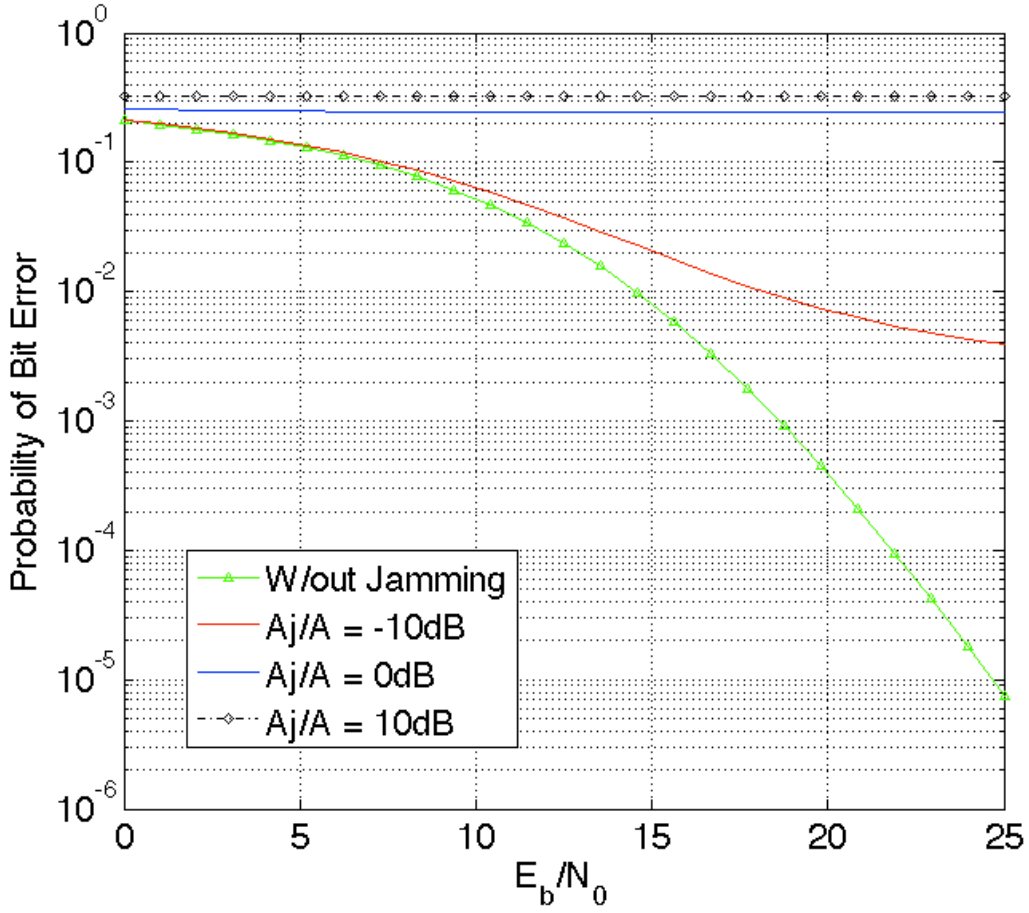

Figure 26. The BER for a MIMO (2×2) system with tone jamming and without complex spreading are shown.

In Figure 26, the BER curves versus E_b / N_0 as function of increasing A_J / A are shown for a MIMO (2×2) system without spreading. As expected, the BER worsens with increasing jammer power. At $A_J / A = 0$ dB, the BER is almost constant, with only slight improvement at $E_b / N_0 > 8$ dB. At $A_J / A = 10$ dB, the BER is completely constant for the whole E_b / N_0 range presented despite using MIMO scheme.

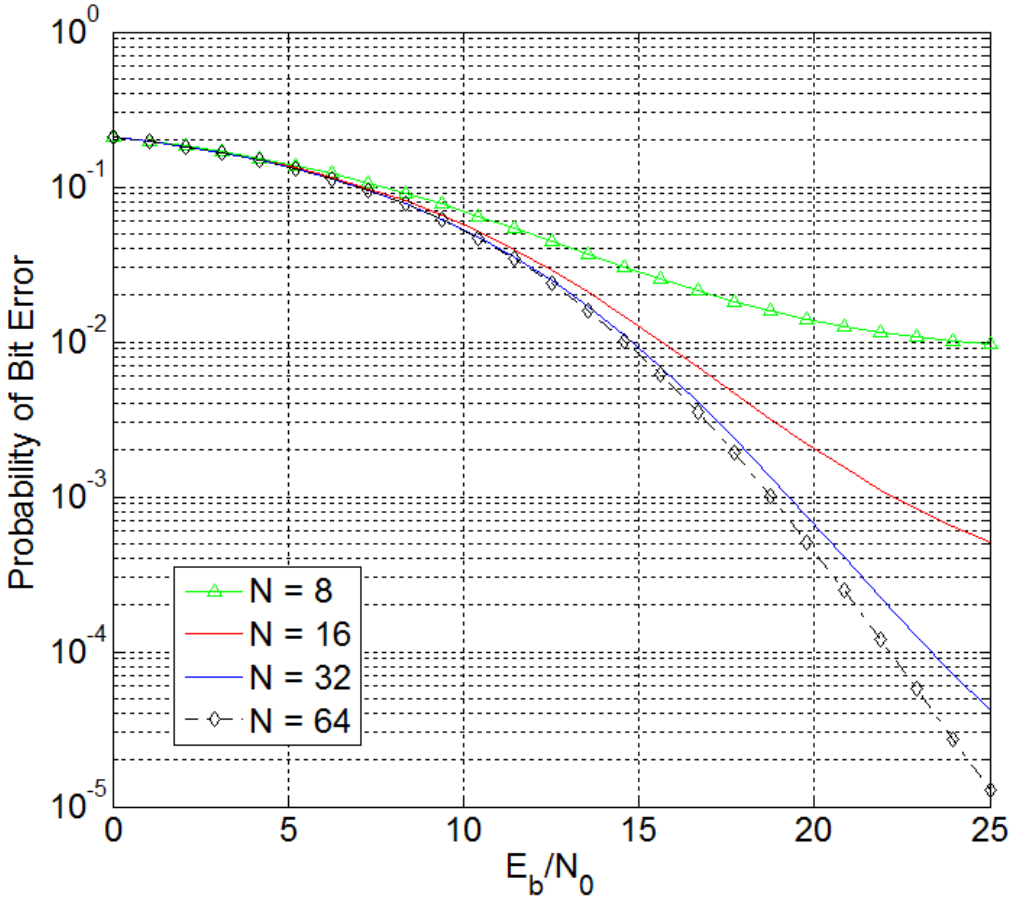

Figure 27. The BER for a MIMO (2×2) system with tone jamming and complex spreading are shown.

For Figure 27, we fix $A_J/A = 0$ dB and apply complex spreading. As expected, the BER improves as N increases. We note the significant improvement from $N = 8$ to $N = 64$.

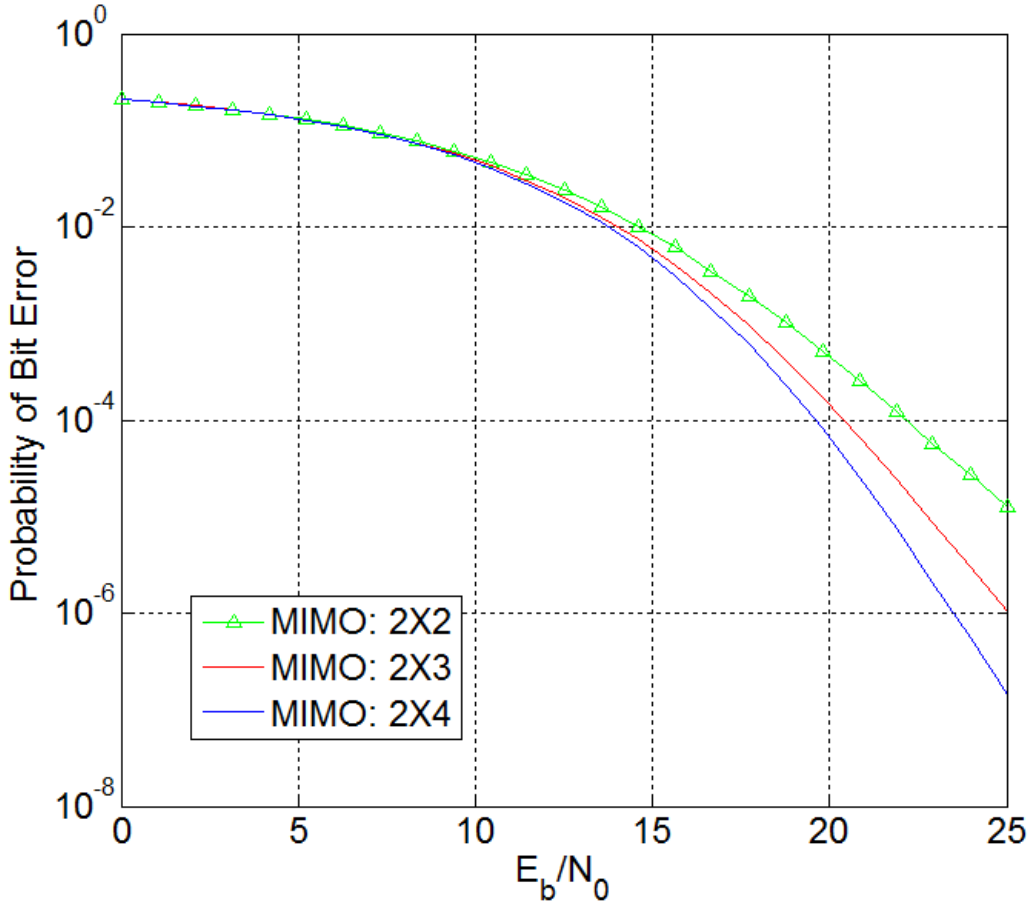

Figure 28. The BER are shown for the following MIMO systems: 2×2, 2×3 and 2×4 with tone jamming.

In Figure 28, we see the improvement of BER as the number of receiver antennas increase. Here we use $L_t = 2$ and $L_r = 2,3,4$ and note the BER improvement as L_r increases for $E_b/N_0 > 15$ dB.

B. SINR AND BER RESULTS FOR MULTI-RATE CHANNEL

The BER are reported using the results in Chapter V for multi-rate channels. Here, we show the BER performance of channel two in Chapter V, Section E, where the PN spreading factor is $N/2$. From the results in Chapter V, it is clear that the BER with jamming is affected.

1. BER for Barrage Jamming

The noise variance corresponding to the result in (5.65) is given by $2\sigma_n^2$, and the jamming variance which corresponds to the result in (5.74) is expressed as $2\sigma_B^2/N = J_0/N$, where $J_0^{'} = 2J_0/N$. Thus, μ in (6.5) is modified to

$$\mu = \sqrt{\frac{1}{\frac{62}{15}L_t(2)\left[\left(\frac{E_b}{N_0}\right)^{-1}+\left(\frac{J_0^{'}}{N_0}\right)^{-1}\frac{1}{N}\right]+1}} = \sqrt{\frac{1}{\frac{124}{15}L_t\left[\left(\frac{E_b}{N_0}\right)^{-1}+\left(\frac{J_0^{'}}{N_0}\right)^{-1}\frac{1}{N}\right]+1}}. \quad (6.10)$$

Hence, (6.10) is used in (6.1) to generate the BER. Here we will show the BER performance for a higher data rate in channel two and compare it with the fixed rate channel BER performance.

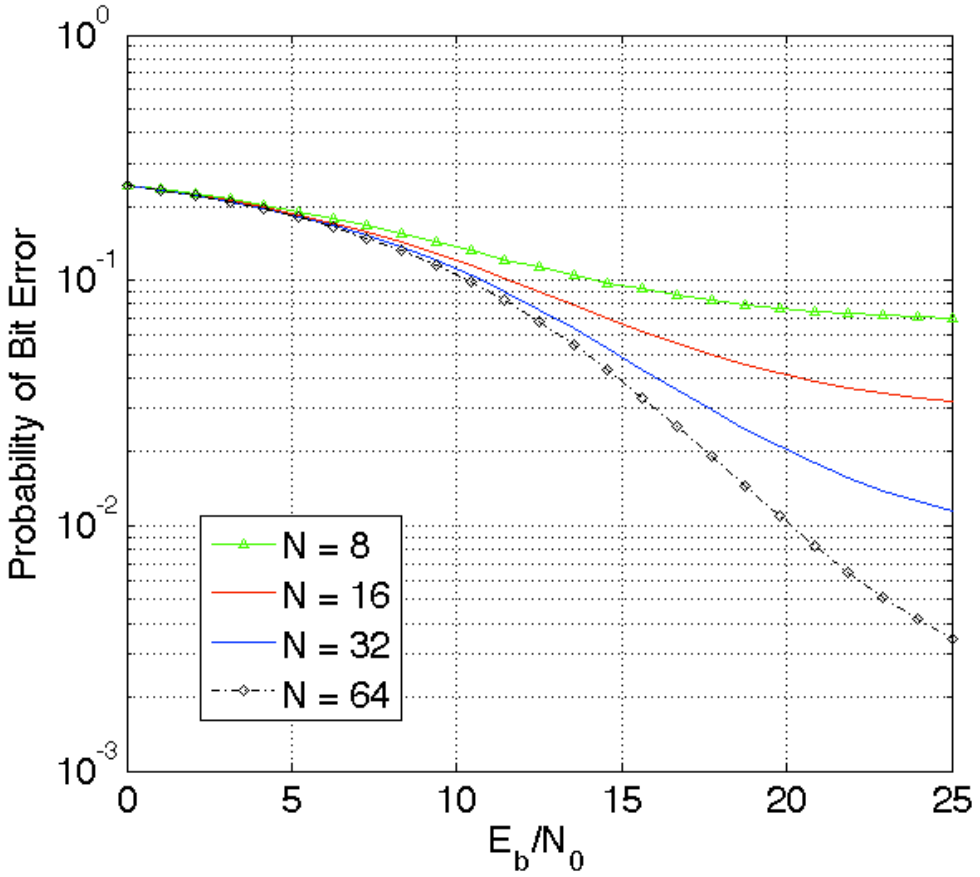

Figure 29. The BER for a MIMO (2×2) system with barrage jamming, complex spreading and $R_s = 2$ are shown.

In Figure 29, the BER for channel two is shown. As expected, the BERs suffer considerable degradation compared to the fixed rate system whose BER results are shown in Figure 21.

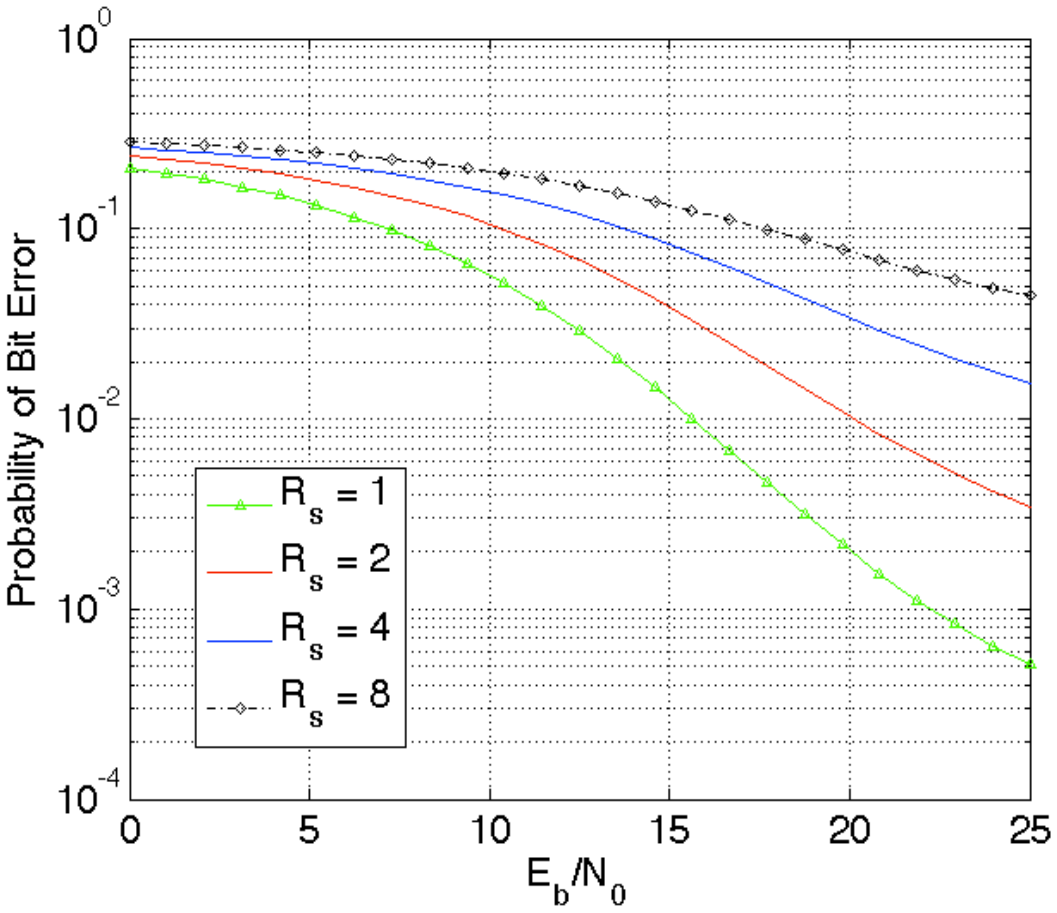

Figure 30. The BER for a MIMO (2×2) system with barrage jamming, complex spreading and increasing R_s are shown.

The effect of increasing the transmission rate is shown in Figure 30. As expected, the BER degrades as the transmission rate increases.

2. BER for Pulsed-Noise Jamming

The jamming variance which corresponds to the result in (5.83) is expressed as $2\sigma_P^2/N$. Thus, μ in (6.6) is modified to

$$\mu = \sqrt{\frac{1}{\frac{62L_t}{15\rho N}(2)\left(\frac{J_0'}{N_0}\right)^{-1}+1}} = \sqrt{\frac{1}{\frac{124L_t}{15\rho N}\left(\frac{J_0'}{N_0}\right)^{-1}+1}}. \tag{6.11}$$

Hence, (6.11) is used in (6.1) to generate the BER results.

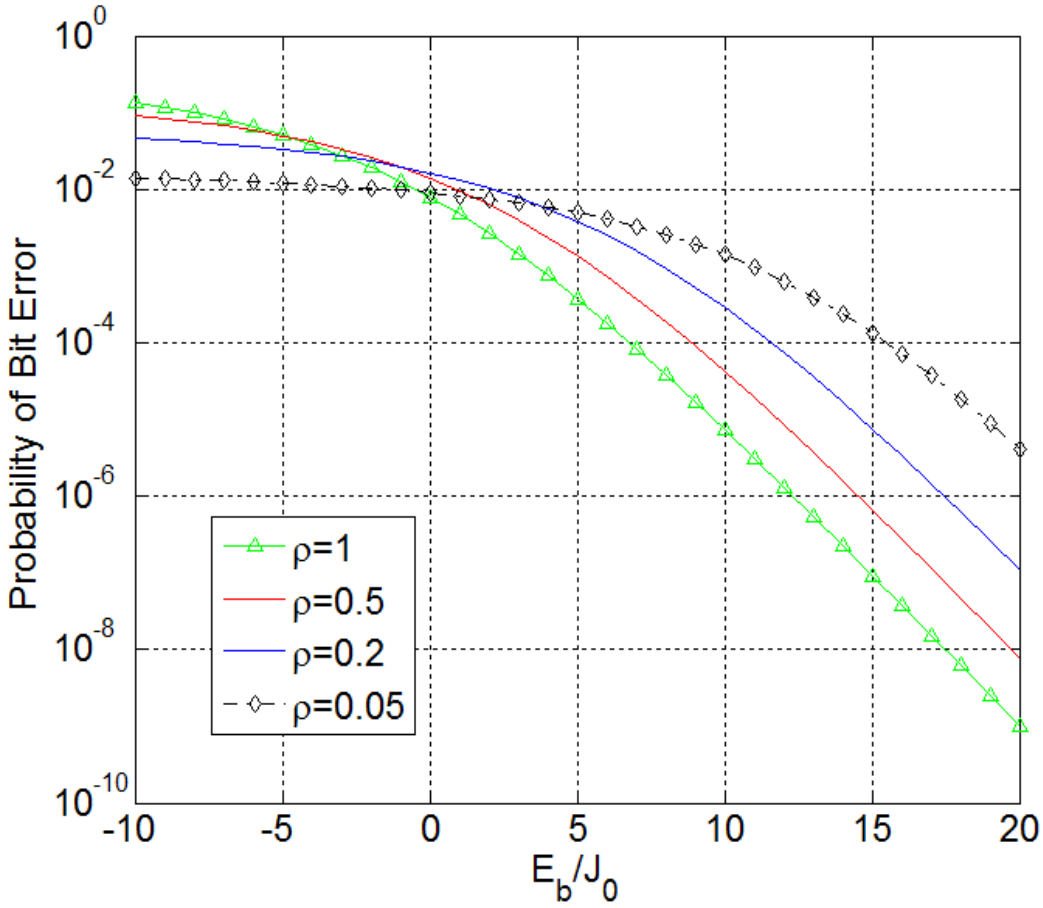

Figure 31. The BER for a MIMO (2×2) system with pulsed-noise jamming (increasing ρ) and $R_s = 2$ are shown.

In Figure 31, the BER curves are shown for a 2×2 MIMO system with $N = 64$ and increasing ρ. It can be seen that the BER is degraded as compared to the BERs shown in Figure 25 since the transmission rate is doubled.

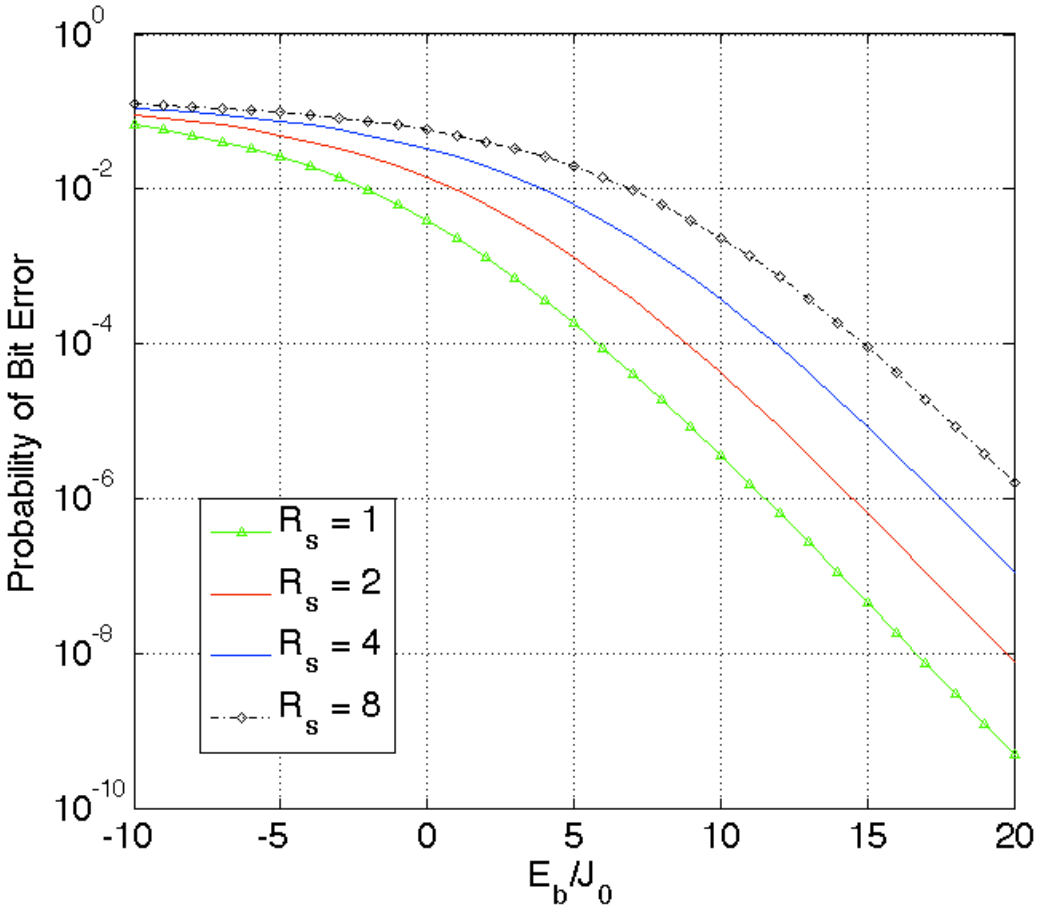

Figure 32. The BER for a MIMO (2×2) system with pulsed-noise jamming of $\rho = 0.5$ and increasing R_s are shown.

In Figure 32, the BER curves are shown for $\rho = 0.5$ as a function of increasing the transmission rate. As expected, the BERs degrade as the transmission rate increases.

3. BER for Tone Jamming

The jamming variance which corresponds to the result in (5.89) is expressed as $\alpha^2\sigma_P^2/2N$. Thus, μ in (6.9) is modified to

$$\mu = \sqrt{\frac{1}{\frac{62L_t}{15}(2)\left[\left(\frac{E_b}{N_0}\right)^{-1} + \left(\frac{A^2}{A_J^2}\right)^{-1}\frac{\alpha^2}{N^2}\right] + 1}} = \sqrt{\frac{1}{\frac{124L_t}{15}\left[\left(\frac{E_b}{N_0}\right)^{-1} + \left(\frac{A^2}{A_J^2}\right)^{-1}\frac{\alpha^2}{N^2}\right] + 1}} . \quad (6.12)$$

Hence, (6.12) is used in (6.1) to generate the BER results.

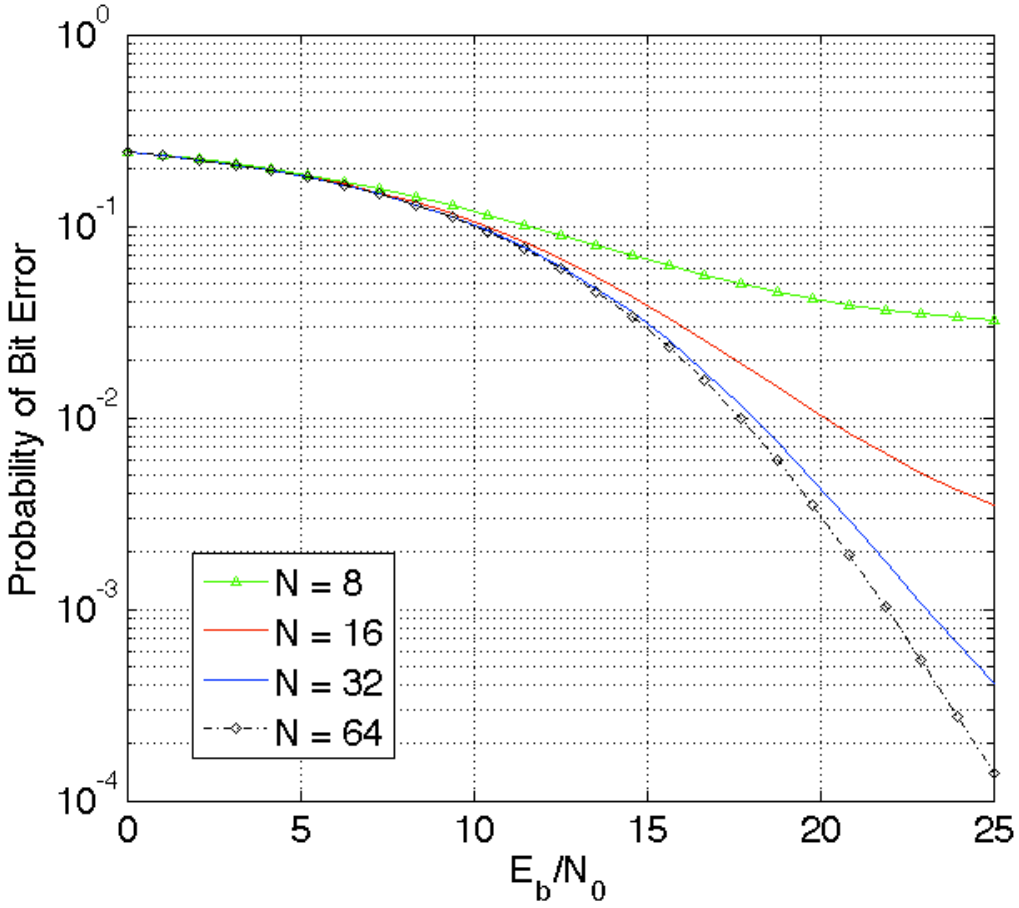

Figure 33. The BER for a MIMO (2×2) system with tone jamming, complex spreading and R_s = 2 are shown.

In Figure 33, the BER curves for channel two are shown. As expected the BERs suffer considerable degradation as compared to the fixed rate system whose corresponding BER curves are shown in Figure 27.

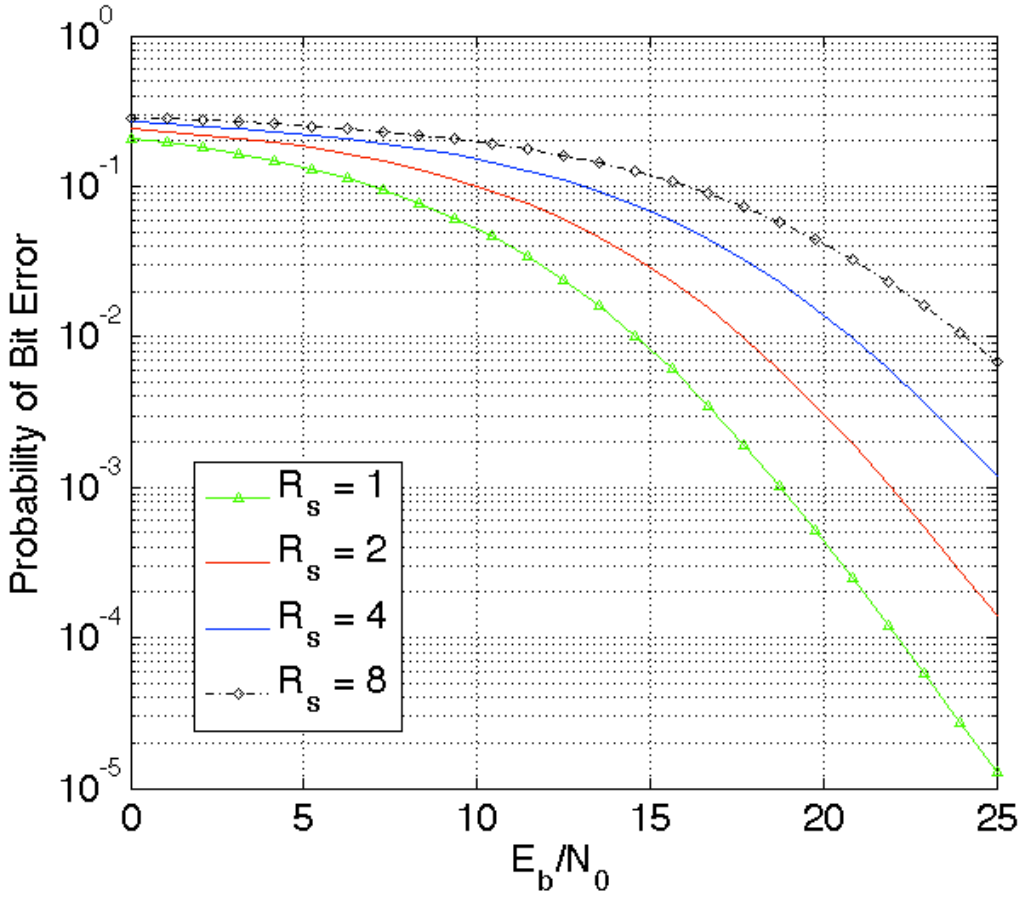

Figure 34. The BER for a MIMO (2×2) system with tone jamming, complex spreading and increasing R_s are shown.

In Figure 34, the BER curves are shown as a function of increasing transmission rate. As expected, the BERs degrade as the transmission rate increases.

C. SUMMARY

In this chapter, we discussed the BER performance of complex spreading MIMO system with orthogonal covering for fixed and multi-rate data transmission. The performance expression for 32QAM was derived with the use of MIMO and SS. While the use of MIMO and SS greatly improves the BER performances, it was shown that the

BER degrades as the transmission rate increases. This is due to the reduction of the spreading factor by a factor of 2^l. Equivalently, the variances of both the noise and jamming are increased by 2^l.

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VII. SUMMARY, CONCLUSIONS AND FUTURE WORK

A. SUMMARY

The general signal model background and the techniques used in this thesis were discussed in Chapter II. The basic illustrations for signaling techniques, which were further discussed in detail in the later chapters, were provided. The types of jamming were also covered.

The procedures for integrated spreading for fixed data rate channels were discussed in Chapter III. The use of repeated Walsh codes/vector is introduced. The PN spreading factor is chosen such that $N = bM$, hence, the Walsh vector is repeated b times to meet the common rate. The procedure for multiuser application that explains the orthogonal covering process were also covered.

The procedure for multi-rate channels was presented in Chapter IV. The concatenation of Walsh vector was explained. The higher data rate channel is assigned a shorter Walsh code, and a lower data rate channel is assigned a longer Walsh code. Hence, to perform orthogonal covering, the Walsh vector in the higher data rate channel must be concatenated to meet the same chip rate as that of the lower data rate channel. It was also shown that the PN spreading factor N is reduced due to the concatenation; i.e., the effective spreading gain is $N/2^l$.

In Chapter V, the fading, noise and jamming interferences were discussed. The noise and jamming covariances were calculated for both fixed and multi-rate channels. It was shown that for the higher data rate channel, the variances of noise and jamming are also larger.

In Chapter VI, the BER performances for both the fixed and multi-rate channels were presented. It was shown that the BER improves with the use of complex spreading and MIMO. It was also shown that the BER degrades as the data rate increases due to the reduction of the PN spreading gain.

B. CONCLUSIONS

The main scope of this thesis was to model and analyze the performance of a complex spreading MIMO system with orthogonal covering in interference and fading. The integrated spreading and de-spreading of the MIMO signal was modeled using a discrete time formulation. The concepts to achieve the complex spread spectrum signaling with orthogonal covering for fixed and multi-rate channels were provided.

The procedures for MIMO signal employing fixed and variable length orthogonal covering were shown using numerical examples. For multi-rate transmission, the concatenation of Walsh vectors is performed in order to achieve orthogonal covering to allow channeling of different data rates. The data rate factor has to be a power of two. For higher transmission rates, the PN spreading gain for each symbol is reduced by a factor of a power of two, depending on the data rate. This results in BER degradation for the higher data rate channels. The use of MIMO, complex spreading and orthogonal covering improve system performance for fading and interference over multi-rate channels.

C. FUTURE WORK

Future work may include forward error correction coding prior to Walsh encoding, complex spreading, and MIMO coding.

APPENDIX

Received signal via Alamouti code for fixed and variable rate transmissions.

Fixed Rate

If $\mathbf{D} = \begin{bmatrix} D_1 & D_2 \end{bmatrix}$, then the transmit signal via Alamouti coding is given by

$$\begin{aligned} \mathbf{X} = \mathbf{WS} &= \frac{\mathbf{W}\boldsymbol{c}}{\|\boldsymbol{c}\|}\mathbf{D} \\ &= \begin{bmatrix} \tilde{w}_1 c_1 D_1 & \tilde{w}_1 c_1 D_2 \\ \tilde{w}_2 c_2 D_1 & \tilde{w}_2 c_2 D_2 \\ \vdots & \vdots \\ \tilde{w}_N c_N D_1 & \tilde{w}_N c_N D_2 \end{bmatrix} \xrightarrow{Alamouti} \mathbf{G} = \begin{bmatrix} \boldsymbol{G}_1 \\ \boldsymbol{G}_2 \\ \vdots \\ \boldsymbol{G}_N \end{bmatrix}, \end{aligned} \tag{A.1}$$

where $\mathbf{W}$ is given in (3.5), and each sub matrix $\boldsymbol{G}_i$ is expressed as

$$\boldsymbol{G}_i = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} \tilde{w}_i c_i D_1 & \tilde{w}_i c_i D_2 \\ -\tilde{w}_i c_i^* D_2^* & \tilde{w}_i c_i^* D_1^* \end{bmatrix}. \tag{A.2}$$

The Walsh chip $\tilde{w}_i \in \{w_1, w_2, \cdots, w_N\}$ is the *ith* diagonal element of $\mathbf{W}$, where $\tilde{w}_j, j = 1,2,\cdots,N$ is given in (3.5). The sum of the channel-tap weighted code words for each sub matrix is

$$\begin{aligned} \boldsymbol{G}_i \xrightarrow{channel} \boldsymbol{G}_i\boldsymbol{h} &= \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} \tilde{w}_i c_i D_1 & \tilde{w}_i c_i D_2 \\ -\tilde{w}_i c_i^* D_2^* & \tilde{w}_i c_i^* D_1^* \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\ &= \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1\tilde{w}_i c_i D_1 + h_2\tilde{w}_i c_i D_2 \\ h_2\tilde{w}_i c_i^* D_1^* - h_1\tilde{w}_i c_i^* D_2^* \end{bmatrix} \end{aligned} \tag{A.3}$$

and can be mapped into $\mathbf{H}\mathbf{X}^{(i)^t}$ where $\mathbf{X}^{(i)}$ is the *ith* row of $\mathbf{X}$ by changing the second element of $\boldsymbol{G}_i\boldsymbol{h}$ into its complex conjugate. Thus, it can be expressed in the form

$$\boldsymbol{G}_i\boldsymbol{h} \xrightarrow{mapped} \mathbf{H}\mathbf{X}^{(i)^t} = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} \tilde{w}_i c_i D_1 \\ \tilde{w}_i c_i D_2 \end{bmatrix}. \tag{A.4}$$

It can be shown that

$$\mathbf{G}\boldsymbol{h} \longrightarrow \mathbf{H}\mathbf{X}^t = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} \tilde{w}_1 c_1 D_1 & \tilde{w}_2 c_2 D_1 & \cdots & \tilde{w}_N c_N D_1 \\ \tilde{w}_1 c_1 D_2 & \tilde{w}_2 c_2 D_2 & \cdots & \tilde{w}_N c_N D_2 \end{bmatrix}, \tag{A.5}$$

where $\mathbf{X}^t = \begin{bmatrix} \mathbf{X}^{(1)^t} & \mathbf{X}^{(2)^t} & \cdots & \mathbf{X}^{(N)^t} \end{bmatrix}$. Hence,

$$\mathbf{X}\mathbf{H}^t = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} \tilde{w}_1 c_1 D_1 & \tilde{w}_1 c_1 D_2 \\ \tilde{w}_2 c_2 D_1 & \tilde{w}_2 c_2 D_2 \\ \vdots & \vdots \\ \tilde{w}_N c_N D_1 & \tilde{w}_N c_N D_2 \end{bmatrix}\begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}. \tag{A.6}$$

Variable Rate

If the multi-rate channel symbol is $\mathbf{D} = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 \end{bmatrix}$, then the concatenated transmit signal $\mathbf{X}$ via Alamouti coding is given by

$$\begin{aligned} \mathbf{X} &= \frac{\boldsymbol{C}}{\|\boldsymbol{c}\|}\hat{\mathbf{S}} \\ &= \begin{bmatrix} c_1\tilde{w}_1 D_1 & c_1\tilde{w}_2 D_2 \\ \vdots & \vdots \\ c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_1 & c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_2 \\ c_{\frac{N}{2}+1}\tilde{w}_1 D_3 & c_{\frac{N}{2}+1}\tilde{w}_1 D_4 \\ \vdots & \vdots \\ c_N\tilde{w}_{\frac{N}{2}} D_3 & c_N\tilde{w}_{\frac{N}{2}} D_4 \end{bmatrix} \xrightarrow{Alamouti} \mathbf{G} = \begin{bmatrix} \boldsymbol{G}_{1,1} \\ \vdots \\ \boldsymbol{G}_{\frac{N}{2},1} \\ \boldsymbol{G}_{\frac{N}{2}+1,2} \\ \vdots \\ \boldsymbol{G}_{N,2} \end{bmatrix}, \end{aligned} \tag{A.7}$$

and each sub matrix $\boldsymbol{G}_{i,k}, k = 1,2$ is expressed as

$$\boldsymbol{G}_{i,1}=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_i\widetilde{w}_i D_1 & c_i\widetilde{w}_i D_2 \\ -c_i^*\widetilde{w}_i D_2^* & c_i^*\widetilde{w}_i D_1^* \end{bmatrix} \quad i=1,2,\cdots,\frac{N}{2}$$
$$\boldsymbol{G}_{i,2}=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_i\widetilde{w}_{i-\frac{N}{2}} D_3 & c_i\widetilde{w}_{i-\frac{N}{2}} D_4 \\ -c_i^*\widetilde{w}_{i-\frac{N}{2}} D_4^* & c_i^*\widetilde{w}_{i-\frac{N}{2}} D_3^* \end{bmatrix}. \quad i=\frac{N}{2}+1,\frac{N}{2}+2,\cdots,N \tag{A.8}$$

The sum of the channel-tap weighted code words for each sub matrix is

$$\begin{aligned}\boldsymbol{G}_{i,1}\xrightarrow{channel}\boldsymbol{G}_{i,1}\boldsymbol{h}&=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_i\widetilde{w}_i D_1 & c_i\widetilde{w}_i D_2 \\ -c_i^*\widetilde{w}_i D_2^* & c_i^*\widetilde{w}_i D_1^* \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\ &=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 c_i\widetilde{w}_i D_1+h_2 c_i\widetilde{w}_i D_2 \\ h_2 c_i^*\widetilde{w}_i D_1^*-h_1 c_i^*\widetilde{w}_i D_2^* \end{bmatrix}, \quad i=1,2,\cdots,\frac{N}{2} \\ \boldsymbol{G}_{i,2}\xrightarrow{channel}\boldsymbol{G}_{i,2}\boldsymbol{h}&=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_i\widetilde{w}_{i-\frac{N}{2}} D_3 & c_i\widetilde{w}_{i-\frac{N}{2}} D_4 \\ -c_i^*\widetilde{w}_{i-\frac{N}{2}} D_4^* & c_i^*\widetilde{w}_{i-\frac{N}{2}} D_3^* \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \\ &=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 c_i\widetilde{w}_{i-\frac{N}{2}} D_3+h_2 c_i\widetilde{w}_{i-\frac{N}{2}} D_4 \\ h_2 c_i^*\widetilde{w}_{i-\frac{N}{2}} D_3^*-h_1 c_i^*\widetilde{w}_{i-\frac{N}{2}} D_4^* \end{bmatrix}. \quad i=\frac{N}{2}+1,\frac{N}{2}+2,\cdots,N\end{aligned} \tag{A.9}$$

These sub-matrices can be mapped into $\mathbf{HX}^{(i,k)^t}$ by changing the second element of $\boldsymbol{G}_{i,k}\boldsymbol{h}$ into its complex conjugate. Thus, it can be expressed in the form

$$\begin{aligned}\boldsymbol{G}_{i,1}\boldsymbol{h}\xrightarrow{mapped}\mathbf{HX}^{(i,1)^t}&=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} c_i\widetilde{w}_i D_1 \\ c_i\widetilde{w}_i D_2 \end{bmatrix} \quad i=1,2,\cdots,\frac{N}{2} \\ \boldsymbol{G}_{i,2}\boldsymbol{h}\xrightarrow{mapped}\mathbf{HX}^{(i,2)^t}&=\frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} c_i\widetilde{w}_{i-\frac{N}{2}} D_3 \\ c_i\widetilde{w}_{i-\frac{N}{2}} D_4 \end{bmatrix}, \; i=\frac{N}{2}+1,\frac{N}{2}+2,\cdots,N\end{aligned} \tag{A.10}$$

where $\mathbf{X}^{(i,1)}$ is the *ith* row of $\mathbf{X}, i = 1,2,\cdots,\frac{N}{2}$, and $\mathbf{X}^{(i,2)}$ is the *ith* row of $\mathbf{X}, i = \frac{N}{2}+1,\cdots,N$. It can be shown that

$$\mathbf{G}\boldsymbol{h} \longrightarrow \mathbf{H}\mathbf{X}^t = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} \mathbf{X}^{(1,1)^t} & \mathbf{X}^{(2,1)^t} & \cdots & \mathbf{X}^{(N,2)^t} \end{bmatrix}$$

$$= \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} c_1\tilde{w}_1 D_1 & c_2\tilde{w}_2 D_1 & \cdots & c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_1 & c_{\frac{N}{2}+1}\tilde{w}_1 D_3 & \cdots & c_N\tilde{w}_{\frac{N}{2}} D_3 \\ c_1\tilde{w}_1 D_2 & c_2\tilde{w}_2 D_2 & \cdots & c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_2 & c_{\frac{N}{2}+1}\tilde{w}_1 D_4 & \cdots & c_N\tilde{w}_{\frac{N}{2}} D_4 \end{bmatrix}. \quad \text{(A.11)}$$

Hence,

$$\mathbf{X}\mathbf{H}^t = \frac{1}{\|\boldsymbol{c}\|}\begin{bmatrix} c_1\tilde{w}_1 D_1 & c_1\tilde{w}_2 D_2 \\ \vdots & \vdots \\ c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_1 & c_{\frac{N}{2}}\tilde{w}_{\frac{N}{2}} D_2 \\ c_{\frac{N}{2}+1}\tilde{w}_1 D_3 & c_{\frac{N}{2}+1}\tilde{w}_1 D_4 \\ \vdots & \vdots \\ c_N\tilde{w}_{\frac{N}{2}} D_3 & c_N\tilde{w}_{\frac{N}{2}} D_4 \end{bmatrix}\begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}. \quad \text{(A.12)}$$

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